

Quantum Optics with Atomic Ensembles

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www.qodl.eu



2012-2018



Cold atoms (MOT), Quantum
memories,
Quantum information

2018-2021



Niels Bohr Institutet

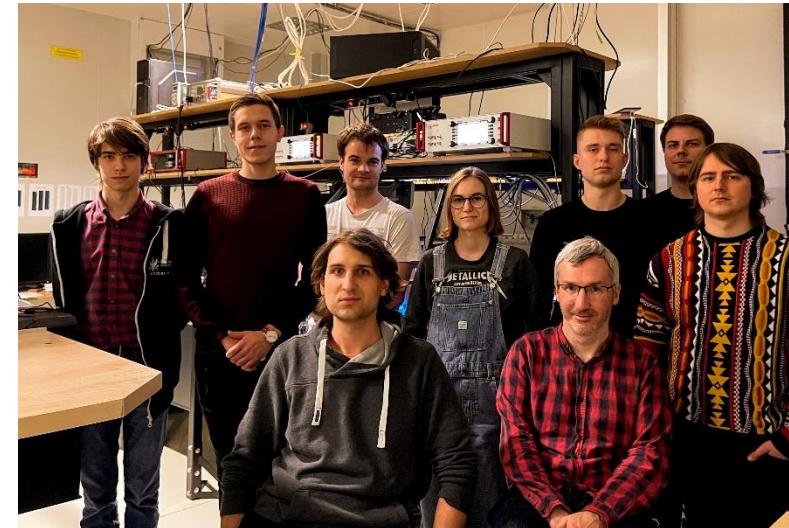
Cavity quantum
optomechanics,
hot atoms

2021-



Rydberg atoms (hot/cold),
analog optical quantum signal
processors (e.g. for
superresolution spectroscopy)

2023-



www.qodl.eu - lab webpage



European Funds
for Smart Economy



Republic
of Poland

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European Union



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Plan for today

- Photons, and photons in many modes
- Two-level atoms
- Three-level atoms, quantum memories
- Rydberg atom ensembles
- My research... ;)

Quantization

$$\vec{D} = \sum_n -p_n(t) \vec{u}_n(\vec{x}) \qquad \vec{B} = \sum_n q_n(t) \vec{v}_n(\vec{x})$$
$$\vec{v}_n = \nabla \times \vec{u}_n$$

$$\dot{q}_n = p_n/\epsilon, \quad \dot{p}_n = -\epsilon\omega_n^2 q_n$$
$$|n_k, n_l, \dots, n_m\rangle = \frac{\hat{a}_k^{\dagger n_k} \hat{a}_l^{\dagger n_l} \cdots \hat{a}_m^{\dagger n_m}}{\sqrt{n_k! n_l! \cdots n_m!}} |0\rangle$$

$$H = \int d^3r \left(\frac{D^2}{2\epsilon} + \frac{B^2}{2\mu} \right) \rightarrow \sum_k \left(\frac{p_k}{2\epsilon} + \frac{\epsilon\omega_k^2 q_k^2}{2} \right)$$

Coherent states

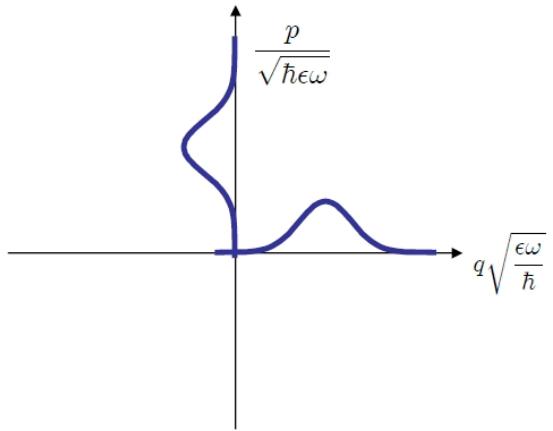
$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

$$|\alpha\rangle = e^{-|\alpha|^2/2} e^{\alpha\hat{a}^\dagger} |0\rangle$$

$$\langle q|\alpha\rangle \propto \exp\left(i\frac{\langle\hat{p}\rangle\hat{q}}{\hbar} - \frac{\epsilon\omega}{\hbar}\frac{(\hat{q}-\langle\hat{q}\rangle)^2}{2}\right)$$

$$\frac{\hat{p}}{\sqrt{\hbar\epsilon\omega}} = \frac{\hat{a} - \hat{a}^\dagger}{i\sqrt{2}}$$

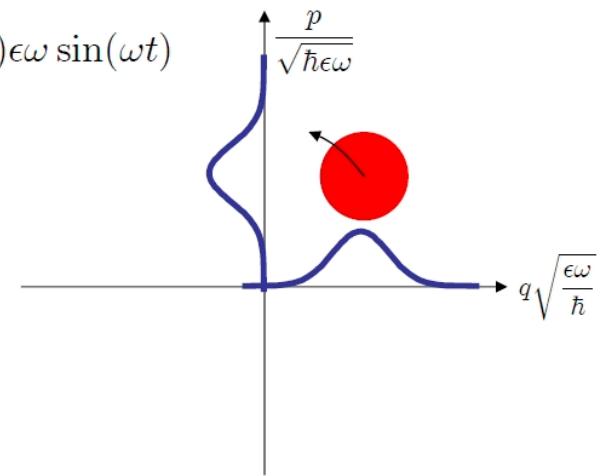
$$\frac{\langle\alpha|\hat{p}|\alpha\rangle}{\sqrt{\hbar\epsilon\omega}} = \frac{\alpha - \alpha^*}{i\sqrt{2}}$$



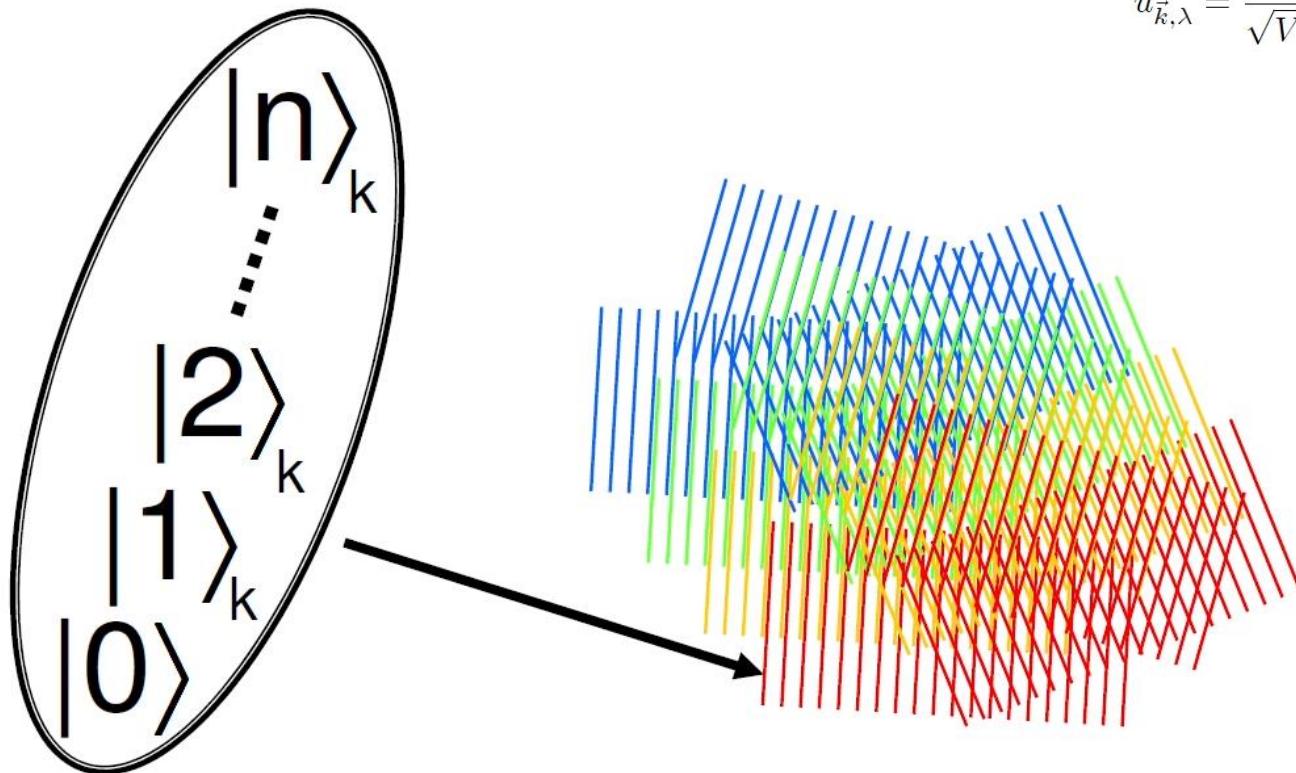
$$H = \hbar\omega(a^\dagger a + 1/2)$$

$$\begin{aligned} q(t) &= q(0) \cos(\omega t) + \frac{p(0)}{\epsilon\omega} \sin(\omega t) \\ p(t) &= p(0) \cos(\omega t) - q(0)\epsilon\omega \sin(\omega t) \end{aligned}$$

$$\hat{a}(t) = \hat{a}(0)e^{-i\omega t}$$



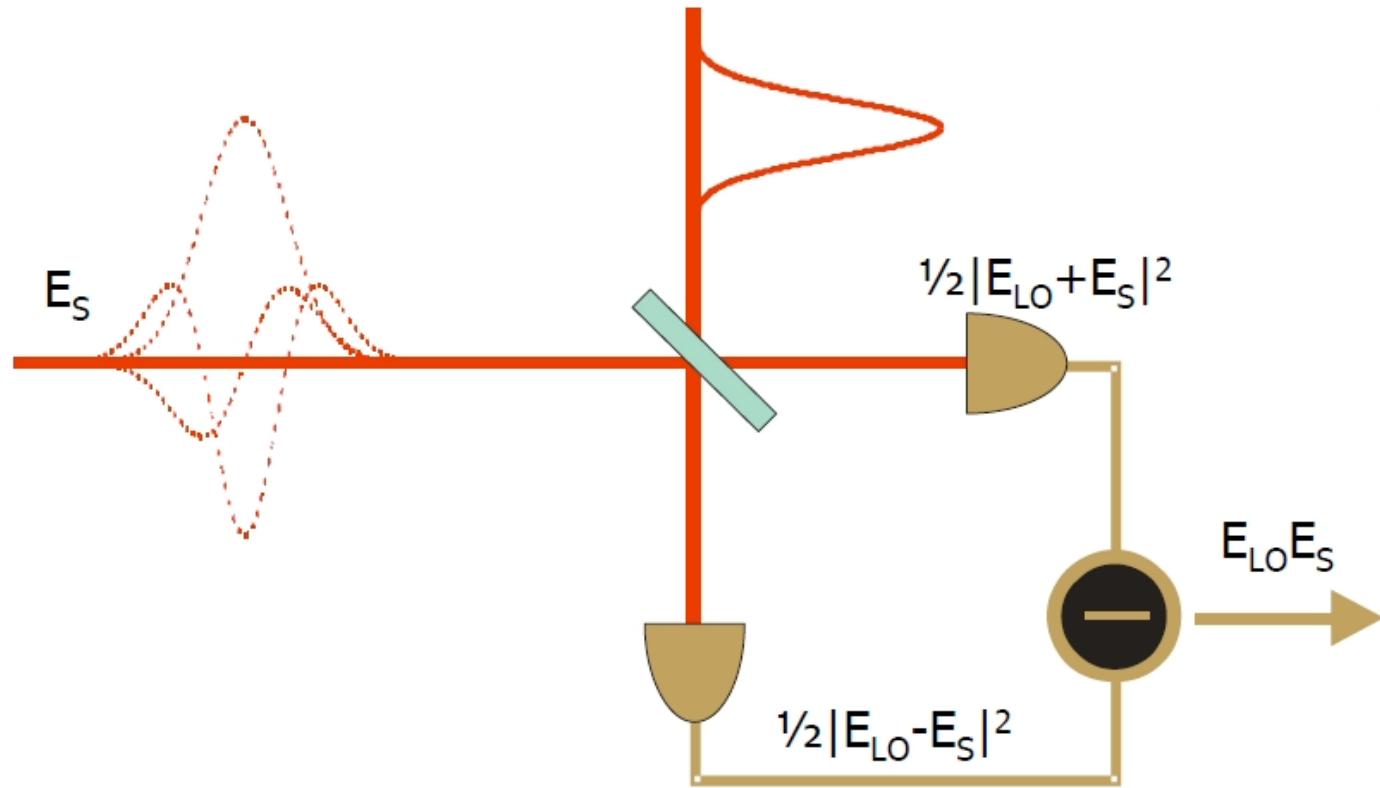
Fields in space and time



$$u_{\vec{k},\lambda} = \frac{1}{\sqrt{V}} \vec{e}_{\vec{k},\lambda} \exp(i\vec{k} \cdot \vec{x})$$

$$\hat{E}(\vec{x}, t) = i \sum_{\vec{k}, \lambda} \sqrt{\frac{\hbar\omega_k}{2\epsilon V}} \hat{a}_{\vec{k}, \lambda} \vec{e}_{\vec{k}, \lambda} \exp(i\vec{k} \cdot \vec{x}) + H.c.$$

Observing fields: homodyne detection



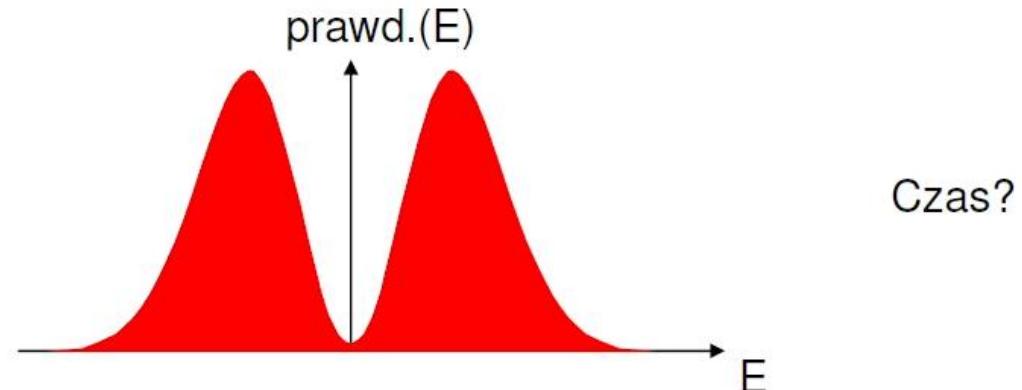
Observing fields: homodyne detection

$$\hat{E} = - \int d^3k \frac{\hat{p}_k(t)}{\epsilon_0} e^{ikr}$$

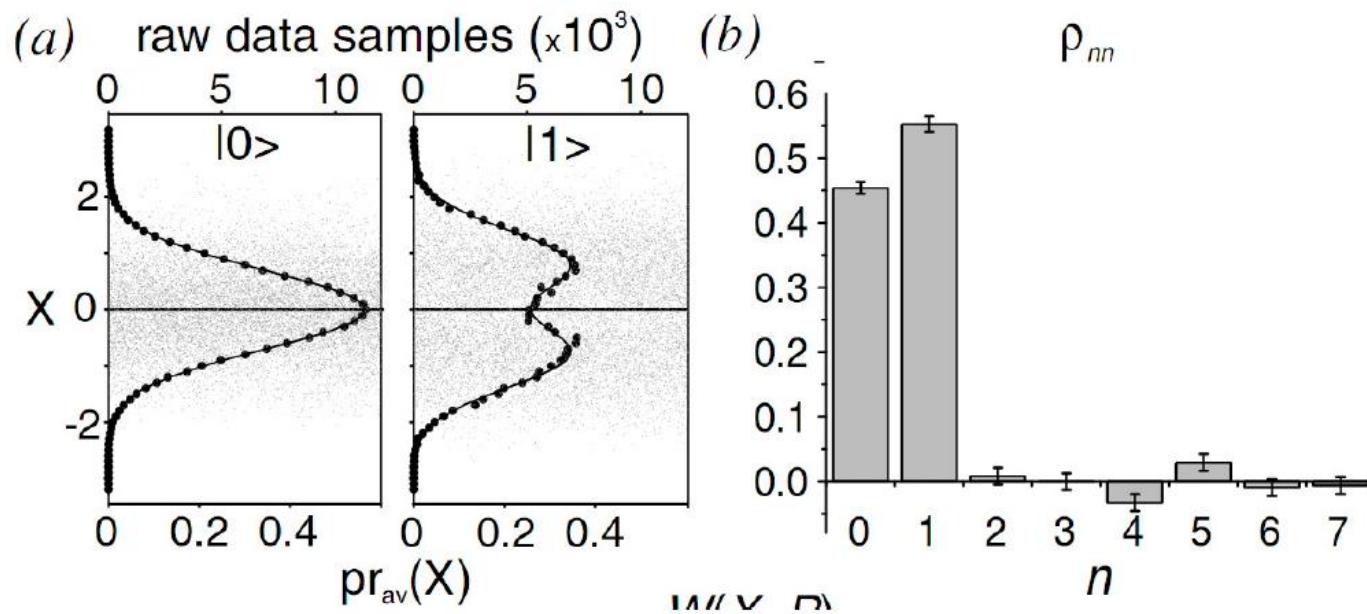
$$\hat{p} = \sqrt{\hbar\epsilon\omega} \frac{\hat{a} - \hat{a}^\dagger}{i\sqrt{2}}$$

$$\hat{a} = \sqrt{\frac{\epsilon\omega}{2\hbar}} \left(\hat{q} + i \frac{\hat{p}}{\epsilon\omega} \right)$$

$$\langle p | 1 \rangle = \sqrt{2} \sqrt[4]{\epsilon\omega\pi\hbar} \exp\left(-\frac{\epsilon\omega p^2}{2\hbar}\right) \sqrt{\frac{\epsilon\omega}{\hbar}} p$$

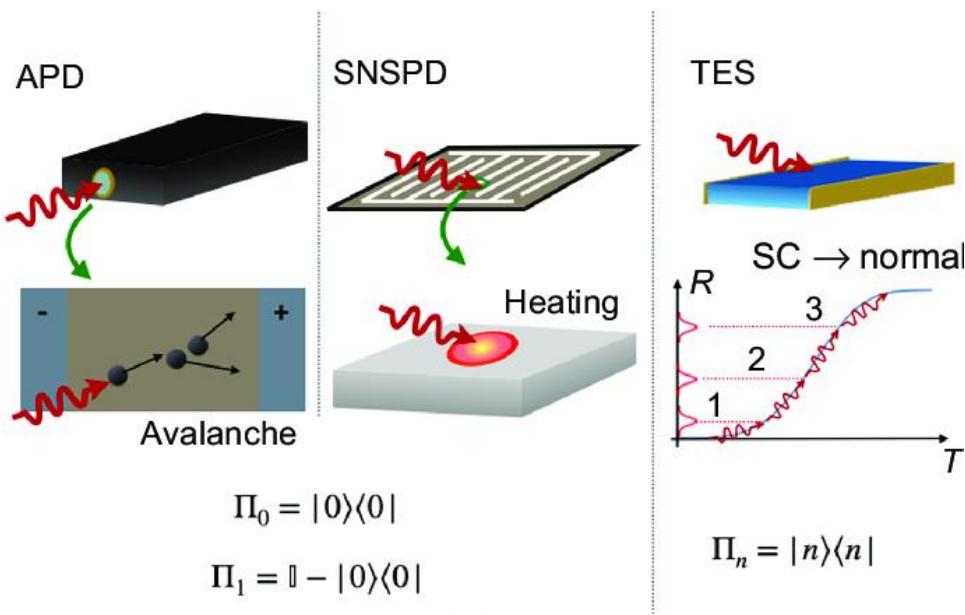


Observing fields: homodyne detection



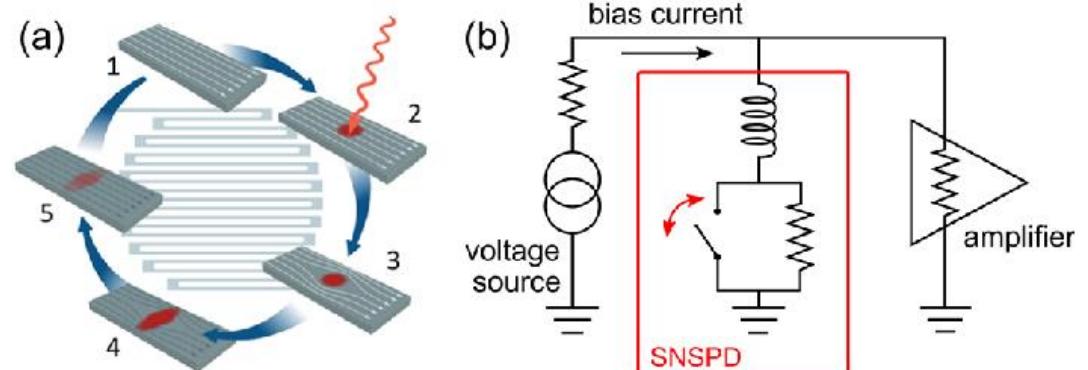
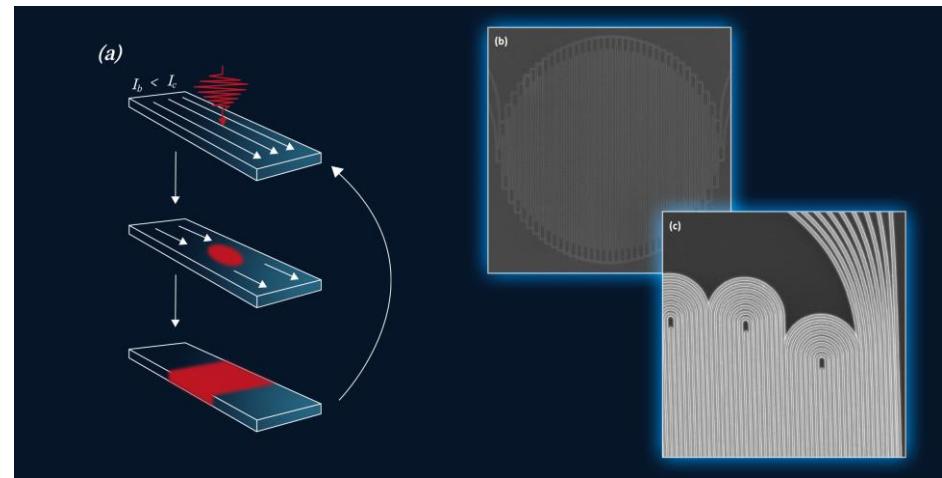
Lvovsky et al., Phys. Rev. Lett. 87, 050402 (2001)

Observing fields: photon counting

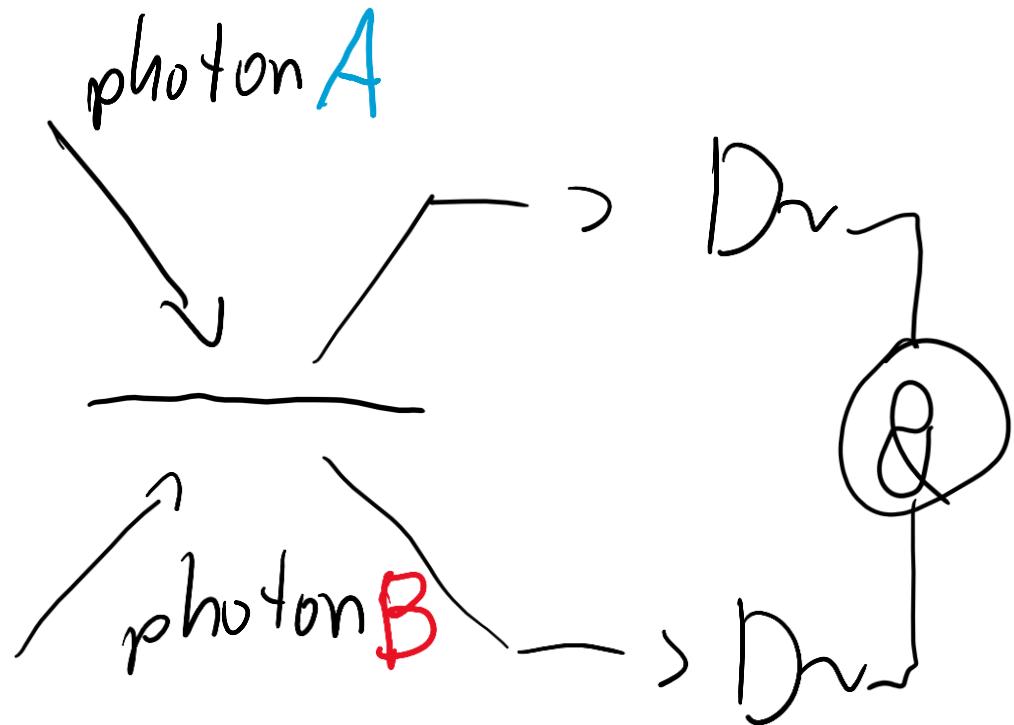


$$\Pi_0 = |0\rangle\langle 0|$$

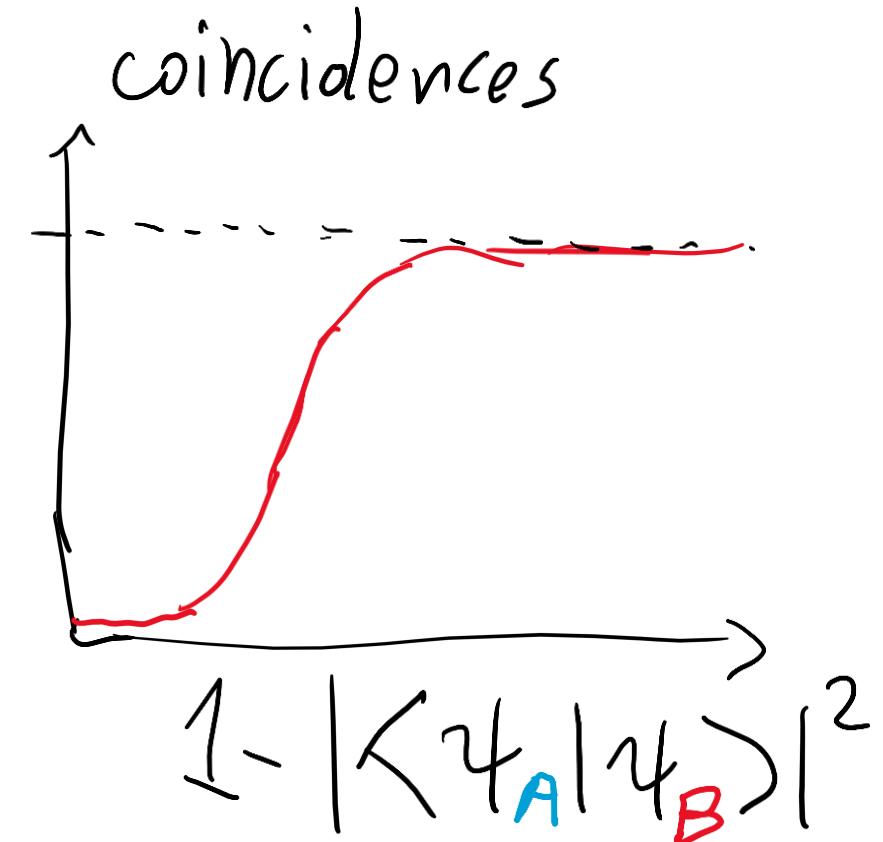
$$\Pi_1 = \mathbb{I} - |0\rangle\langle 0|$$



Interference: Hong-Ou-Mandel



$$\text{BS matrix } x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$



Propagation through medium

$$\Delta \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2}$$

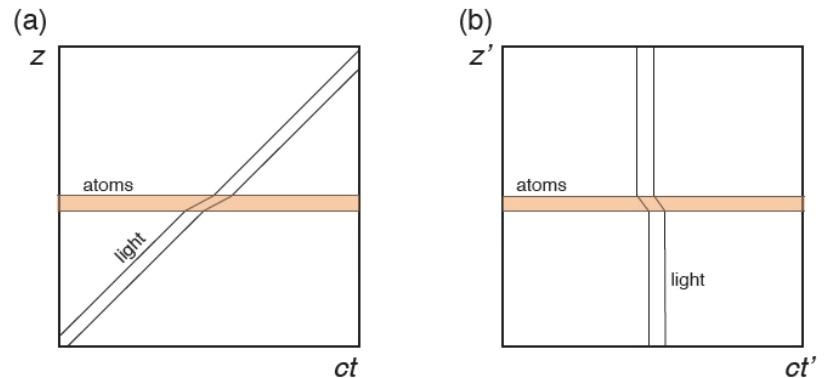
$$\left| \frac{\partial^2 \mathcal{P}^{NL}}{\partial t^2} \right| \ll \omega_0 \left| \frac{\partial \mathcal{P}^{NL}}{\partial t} \right| \ll \omega_0^2 |\mathcal{P}^{NL}|$$

$$\left| \frac{\partial^2 \mathcal{E}}{\partial t^2} \right| \ll \omega_0 \left| \frac{\partial \mathcal{E}}{\partial t} \right| \ll \omega_0^2 |\mathcal{E}|$$

$$\left| \frac{\partial^2 \mathcal{E}}{\partial z^2} \right| \ll k_0 \left| \frac{\partial \mathcal{E}}{\partial z} \right| \ll k_0^2 |\mathcal{E}|$$

$$\Delta_{\perp} \mathcal{E} + 2ik_0 \frac{\partial}{\partial z} \mathcal{E} + 2i \frac{(1+\chi)\omega_0}{c^2} \frac{\partial}{\partial t} \mathcal{E} = -\frac{\omega_0^2}{\epsilon_0 c^2} \mathcal{P}^{NL}$$

$$\Delta_{\perp} \mathcal{E} + 2ik_0 \frac{\partial}{\partial z} \mathcal{E} = -\frac{\omega_0^2}{\epsilon_0 c^2} \mathcal{P}^{NL}$$



$$\mathbf{E}(\mathbf{r},t) = \bar{\mathbf{E}}(\mathbf{r},t) + c.c. = \frac{1}{2} \mathcal{E}(\mathbf{r},t) e^{i(k_0 z - \omega_0 t)} + c.c.$$

$$\mathbf{P}^{NL}(\mathbf{r},t) = \bar{\mathbf{P}}^{NL}(\mathbf{r},t) + c.c. = \frac{1}{2} \mathcal{P}^{NL}(\mathbf{r},t) e^{i(k_0 z - \omega_0 t)} + c.c.$$

$$\mathbf{P} = \frac{1}{V} \text{Tr} \left(\bigotimes_i^N \hat{\rho}_i \sum_j^N \hat{\mathbf{d}}^{(j)} \right) = \frac{1}{V} \sum_j^N \prod_i^N \text{Tr}(\hat{\rho}_i \hat{\mathbf{d}}^{(j)}) = \frac{N}{V} \text{Tr}(\hat{\rho} \hat{\mathbf{d}}),$$

$$\boxed{\mathbf{P}(\mathbf{r},t) = n(\mathbf{r}) \text{Tr}(\hat{\rho}(\mathbf{r},t) \hat{\mathbf{d}}),}$$

Atoms: 2 levels, Rabi oscillations

$$\hat{H} = \hat{H}_B + \hat{V}.$$

$$\begin{aligned}\hat{H}_I &= \exp(i\hat{H}_0 t/\hbar)(\hat{H} - \hat{H}_0)\exp(-i\hat{H}_0 t/\hbar) = \\ &= -\hbar\Delta\hat{\sigma}_{ee} - \frac{1}{2}\mathbf{d}_{ge} \cdot (\mathbf{A}e^{-2i\omega t} + \mathbf{A}^*)\hat{\sigma}_{ge} - \frac{1}{2}\mathbf{d}_{eg} \cdot (\mathbf{A}^*e^{2i\omega t} + \mathbf{A})\hat{\sigma}_{eg},\end{aligned}$$

$$\hat{H}_B = \hbar\omega_{ge}|e\rangle\langle e|.$$

$$\hat{H}'_I = -\hbar\Delta|\textcolor{teal}{1}\rangle\langle\textcolor{teal}{1}| - \hbar\frac{\Omega^*}{2}|0\rangle\langle\textcolor{violet}{1}| - \hbar\frac{\Omega}{2}|1\rangle\langle\textcolor{violet}{0}|.$$

$$\hat{V} = -\mathbf{E} \cdot \hat{\mathbf{d}},$$

$$\Omega = \frac{\mathbf{A} \cdot \mathbf{d}_{eg}}{\hbar}$$

$$\hat{V} = -\frac{1}{2}(\mathbf{A} \cdot \mathbf{d}_{ge}e^{-i\omega t} + \mathbf{A}^* \cdot \mathbf{d}_{ge}e^{i\omega t})\hat{\sigma}_{ge} - \frac{1}{2}(\mathbf{A} \cdot \mathbf{d}_{eg}e^{-i\omega t} + \mathbf{A}^* \cdot \mathbf{d}_{eg}e^{i\omega t})\hat{\sigma}_{eg},$$

Steady state: atomic polarization, absorption, dispersion

$$\dot{\hat{\rho}} = \mathcal{L}(\hat{\rho}), \quad \hat{\Lambda} = \Gamma \hat{\sigma}_{gg} \rho_{ee}. \quad \hat{\Gamma} = \Gamma \hat{\sigma}_{ee}.$$

$$\dot{\hat{\rho}} = -\frac{i}{\hbar} [\hat{H}'_I, \hat{\rho}] - \frac{1}{2} \{ \hat{\Gamma}, \hat{\rho} \} + \hat{\Lambda},$$

$$\dot{\hat{\rho}}_S = -\frac{i}{\hbar} [\hat{H}_S, \hat{\rho}_S] + \sum_i \gamma_i (\hat{R}_i \hat{\rho}_S \hat{R}_i^\dagger - \frac{1}{2} \{ \hat{R}_i^\dagger \hat{R}_i, \hat{\rho}_S \}),$$

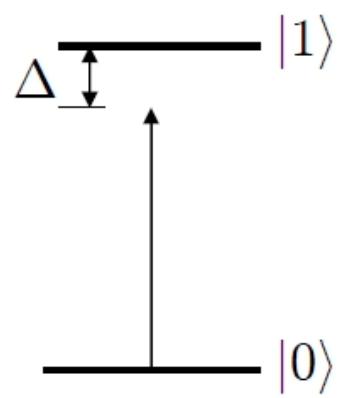
$$\begin{aligned} \dot{\hat{\rho}} = & -\frac{i}{\hbar} [\hat{H}_I, \hat{\rho}] + \gamma_1 (\hat{\sigma}_{ge} \hat{\rho} \hat{\sigma}_{eg} - \frac{1}{2} \{ \hat{\sigma}_{eg} \hat{\sigma}_{ge}, \hat{\rho} \}) + \gamma_2 (\hat{\sigma}_{he} \hat{\rho} \hat{\sigma}_{eh} - \frac{1}{2} \{ \hat{\sigma}_{eh} \hat{\sigma}_{he}, \hat{\rho} \}) + \\ & + \gamma_3 (|g\rangle \langle g| \hat{\rho} |g\rangle \langle g| - \frac{1}{2} \{ |g\rangle \langle g|, \hat{\rho} \}) + \gamma_4 (|h\rangle \langle h| \hat{\rho} |h\rangle \langle h| - \frac{1}{2} \{ |h\rangle \langle h|, \hat{\rho} \}). \end{aligned} \quad (1.12)$$

$$\rho_{ee} = \frac{|\Omega|^2}{\Gamma^2 + 2|\Omega|^2 + 4\Delta^2},$$

$$\rho_{eg} = -\frac{2(\Delta - i\Gamma/2)\Omega}{\Gamma^2 + 2|\Omega|^2 + 4\Delta^2}. \quad \rho_{eg} \approx -\frac{\Omega}{2(\Delta + i\Gamma/2)}.$$

$$\boxed{\mathbf{P}(\mathbf{r},t) = n(\mathbf{r}) \text{Tr}(\hat{\rho}(\mathbf{r},t) \hat{\mathbf{d}}),}$$

Lightshift (AC Stark shift)



$$H = \hbar \begin{pmatrix} \omega_0 & \Omega e^{i\omega t}/2 \\ \Omega^* e^{-i\omega t}/2 & 0 \end{pmatrix}$$

$$H_{int} = \frac{\hbar}{2} \begin{pmatrix} 2\Delta & \Omega \\ \Omega^* & 0 \end{pmatrix}$$

Potraktujmy to jako $\underbrace{\hbar \begin{pmatrix} \Delta & 0 \\ 0 & 0 \end{pmatrix}}_{H_0} + \underbrace{\frac{\hbar}{2} \begin{pmatrix} 0 & \Omega \\ \Omega^* & 0 \end{pmatrix}}_V$

$$E_0 = E_0^{(0)} + \langle 0 | V | 0 \rangle + \frac{|\langle 0 | V | 1 \rangle|^2}{\Delta}$$

\nearrow \nwarrow

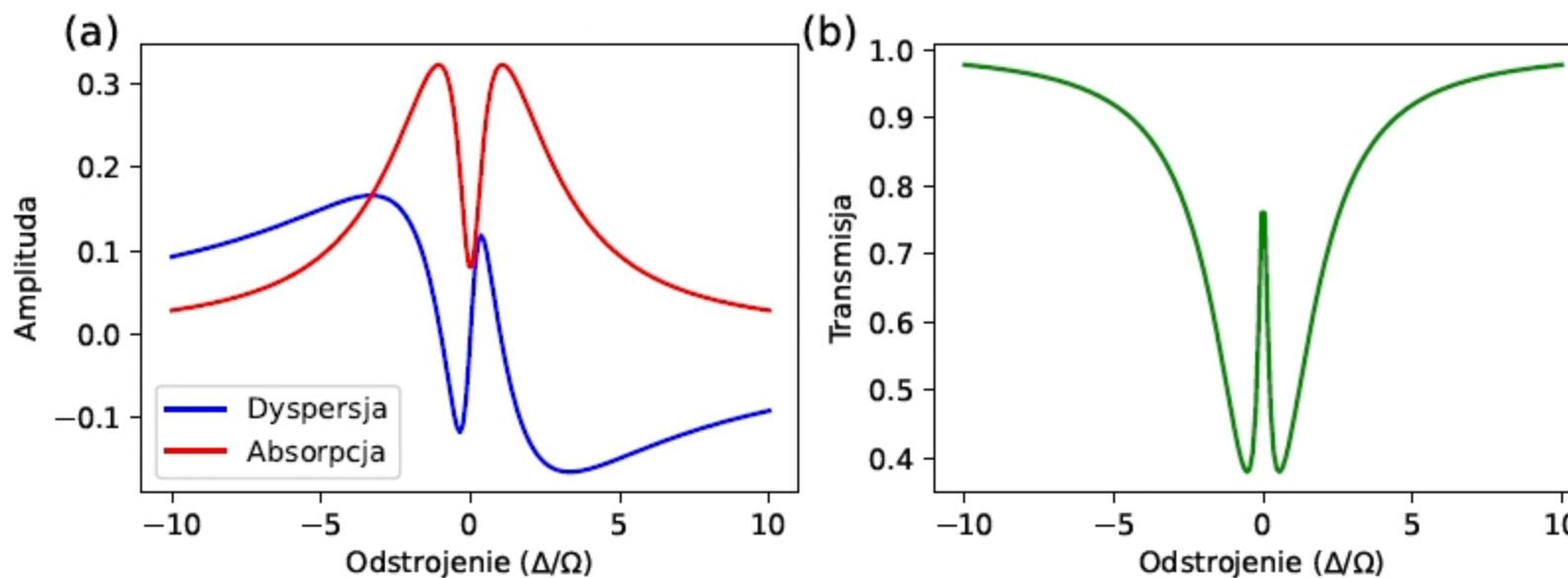
$$\sum_{m \neq 0} \frac{1}{4\hbar} \frac{|Ed_{m0}|^2}{\omega - \omega_{m0}}$$

Three levels: EIT

$$\hat{H}_I = -\frac{\hbar}{2} \begin{pmatrix} 0 & 0 & \Omega_1^* \\ 0 & 2\delta & \Omega_2^* \\ \Omega_1 & \Omega_2 & -2\Delta \end{pmatrix},$$

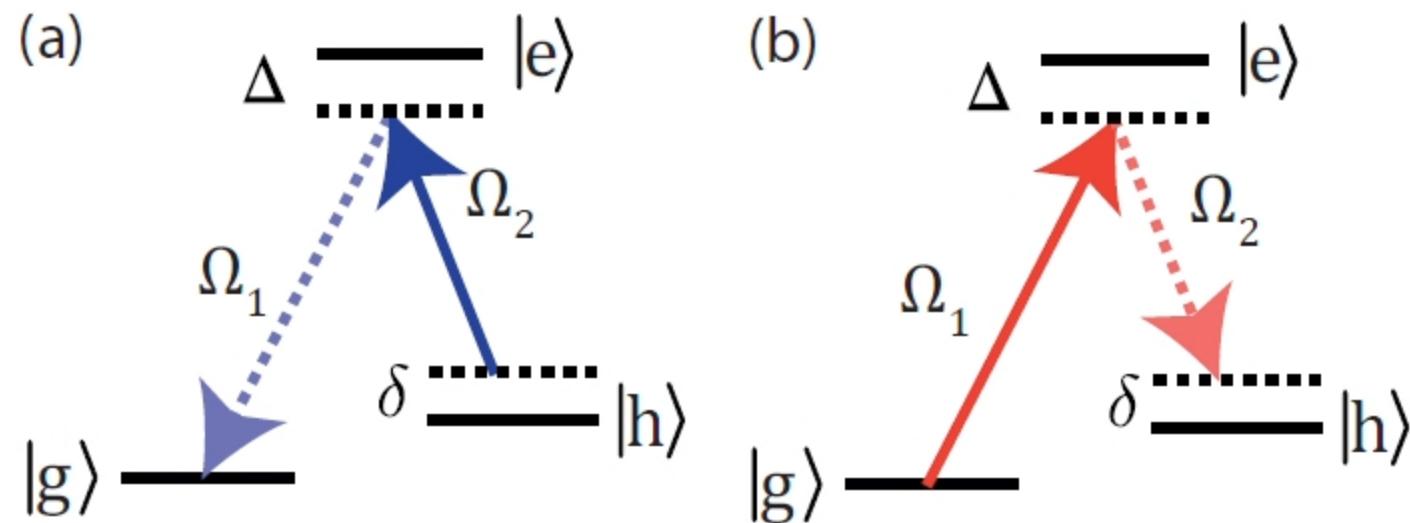
$$\rho_{gr} = -\frac{\Omega_c^* \Omega_p}{(\gamma - i\delta)(\Gamma/4 - i\Delta) + |\Omega_c|^2}$$

$$\chi \Xi = i \frac{nd_{ge}^2}{\varepsilon_0 \hbar} \frac{\gamma - i\delta}{(\gamma - i\delta)(\Gamma/4 - i\Delta) + |\Omega_c|^2}$$



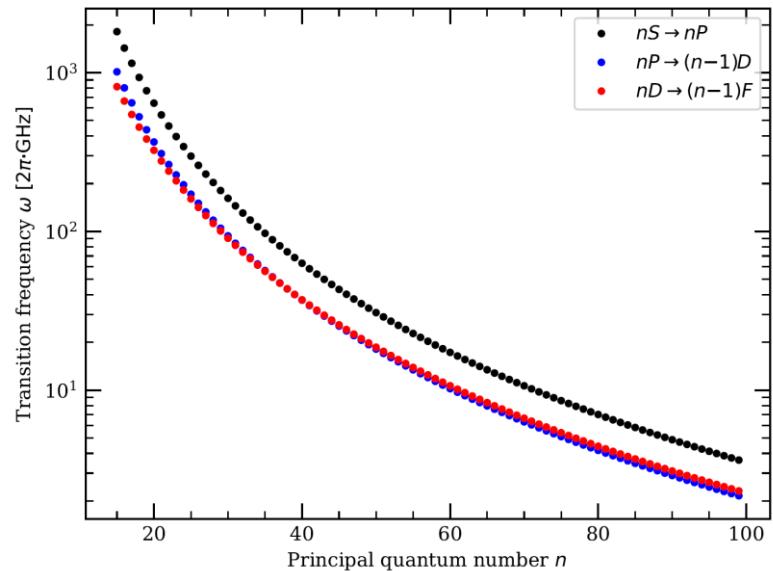
Three levels: two-Photon Rabi oscillation

$$\hat{H}_I = -\frac{\hbar}{2} \begin{pmatrix} 0 & 0 & \Omega_1^* \\ 0 & 2\delta & \Omega_2^* \\ \Omega_1 & \Omega_2 & -2\Delta \end{pmatrix}, \quad \dot{\rho}_{hg} = -\underbrace{\frac{\Omega_1\Omega_2^*}{2\Gamma + 4i\Delta}}_{\text{Raman}} + i \left(\underbrace{\frac{\Delta|\Omega_2|^2}{\Gamma^2 + 4\Delta^2}}_{\text{ac-Stark shift}} + \delta \right) \rho_{hg} - \left(\frac{1}{2}\gamma + \underbrace{\frac{\Gamma|\Omega_2|^2}{2\Gamma^2 + 8\Delta^2}}_{\text{power broadening}} \right) \rho_{hg}. \quad (2.68)$$

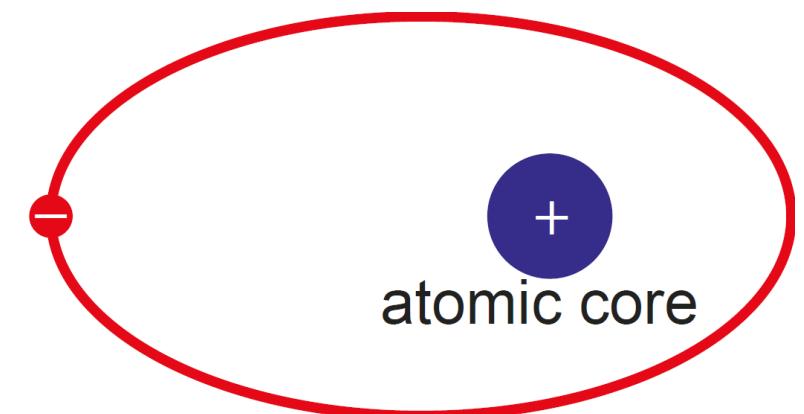


Rydberg atoms

- large principal quantum numbers $n \approx 15-100+$
- long state lifetimes $\tau \approx 15-200 \mu\text{s}$
- long interaction radii $\sim \mu\text{m}$
- microwave (and mm-wave) energy levels transitions $\Delta f \approx 1000-2 \text{ GHz}$



valence
electron



Interacting atoms

$$V(R)_{dd} = \frac{1}{4\pi\varepsilon_0 R^3} \left(\vec{d}_1 \cdot \vec{d}_2 - 3(\vec{d}_1 \cdot \hat{R})(\vec{d}_2 \cdot \hat{R}) \right)$$

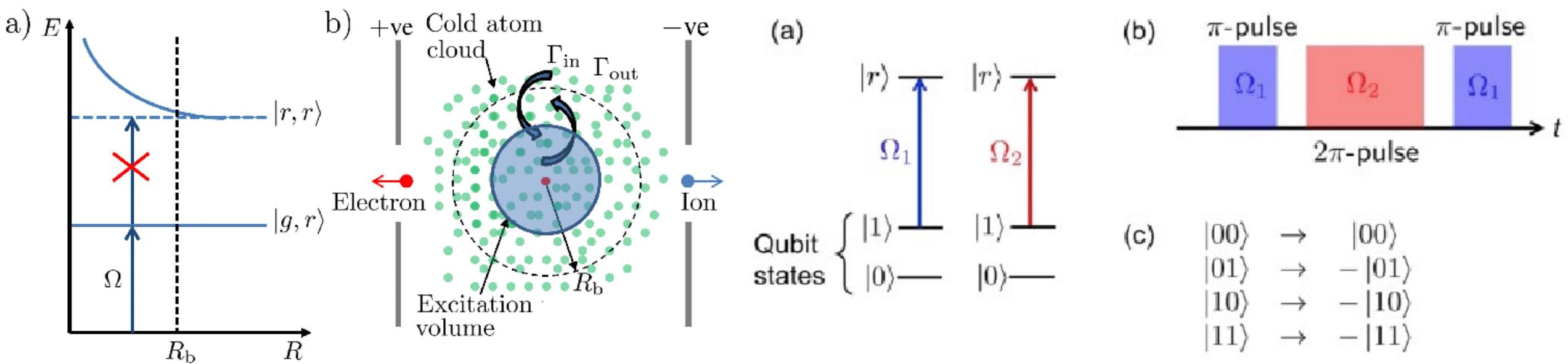
$$\delta E = -\frac{V^2}{\Delta_{r'r''}} \propto -\frac{1}{R^6}, \quad V(R) \ll \Delta_{r'r''}$$

$$\hat{H} = \begin{pmatrix} 0 & V(R) \\ V(R)^* & \Delta_{r'r''} \end{pmatrix}$$

$$\delta E = -V(R) \propto \frac{1}{R^3} \quad V(R) \gg \Delta_{r'r''}$$

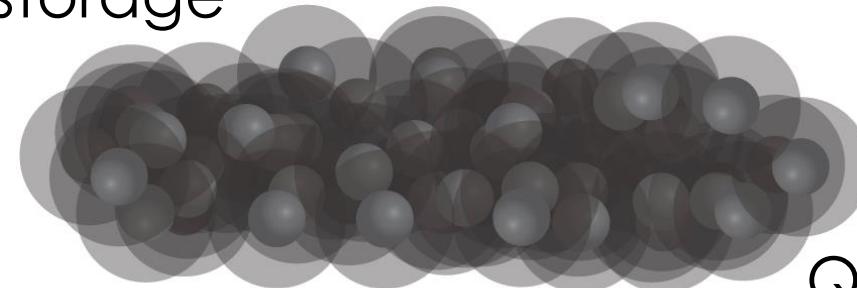
$$\delta E = \frac{\Delta_{r'r''}}{2} - \sqrt{\frac{\Delta_{r'r''}^2}{4} + V(R)^2}$$

Rydberg blockade



Multifunctional quantum memories

Photon spin-wave storage



Photon generation

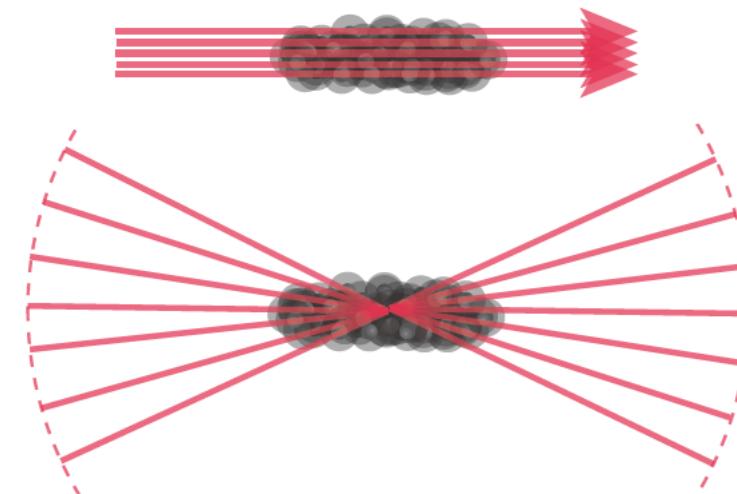
Spin-wave interference

Quantum repeater

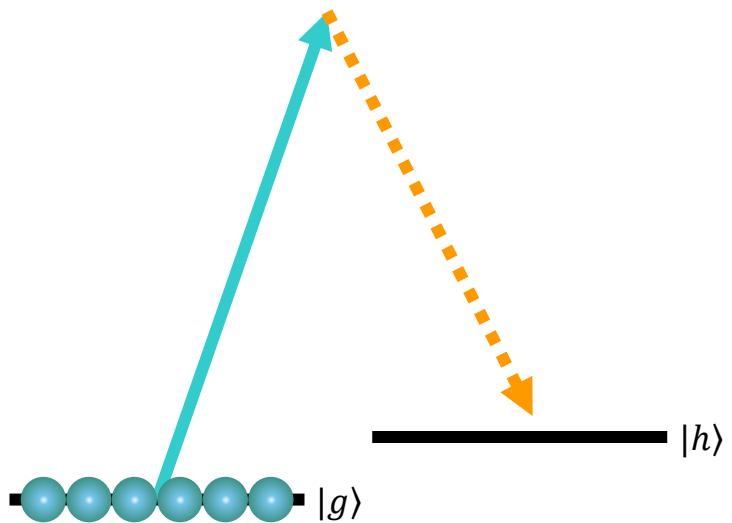
Error correction

Quantum gates: linear, nonlinear

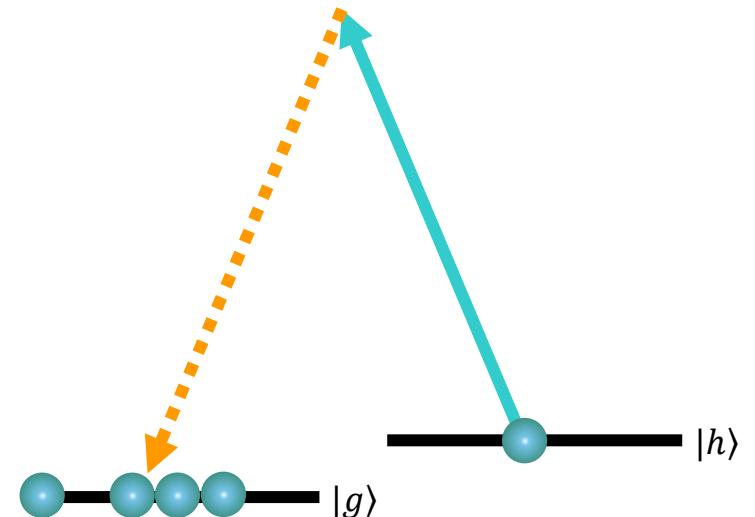
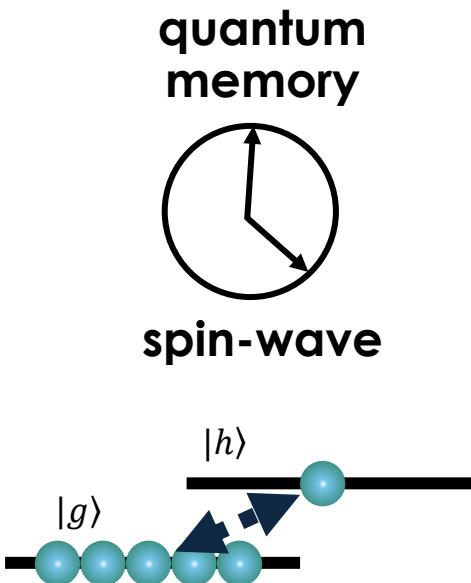
Qudit storage: spatial, temporal



Raman interface

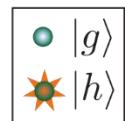


two-mode squeezed state
creation via off-resonant
Raman scattering

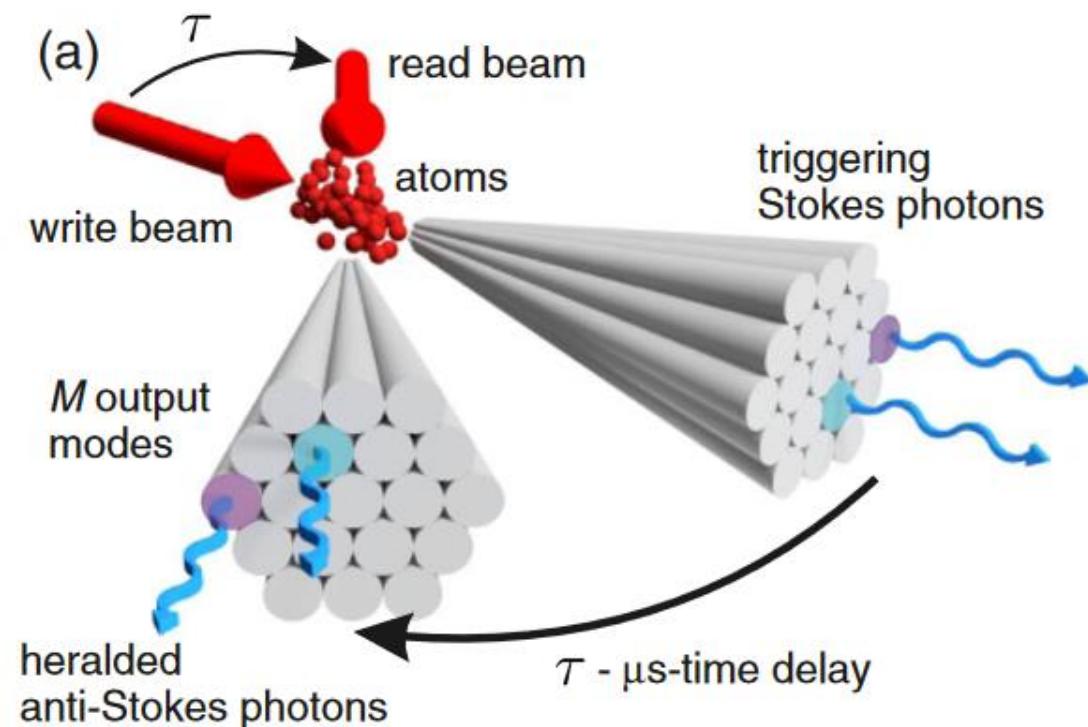
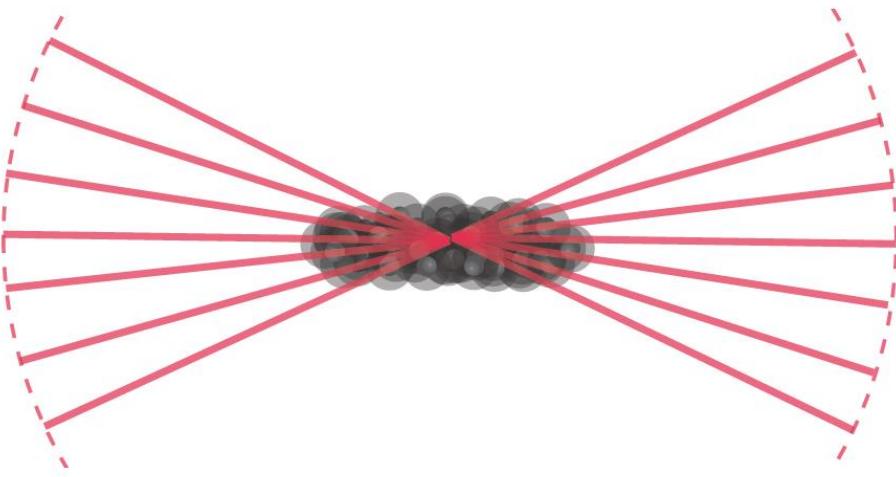


readout stage after storage time
results in **annihilation of spin-wave**

$$\frac{1}{\sqrt{N}} \left(e^{i\mathbf{K} \cdot \mathbf{r}_1} \left| \begin{array}{c} \text{orange star} \\ \text{green circles} \end{array} \right\rangle + e^{i\mathbf{K} \cdot \mathbf{r}_2} \left| \begin{array}{c} \text{green circles} \\ \text{orange star} \end{array} \right\rangle + e^{i\mathbf{K} \cdot \mathbf{r}_3} \left| \begin{array}{c} \text{orange star} \\ \text{green circles} \end{array} \right\rangle + \dots \right)$$

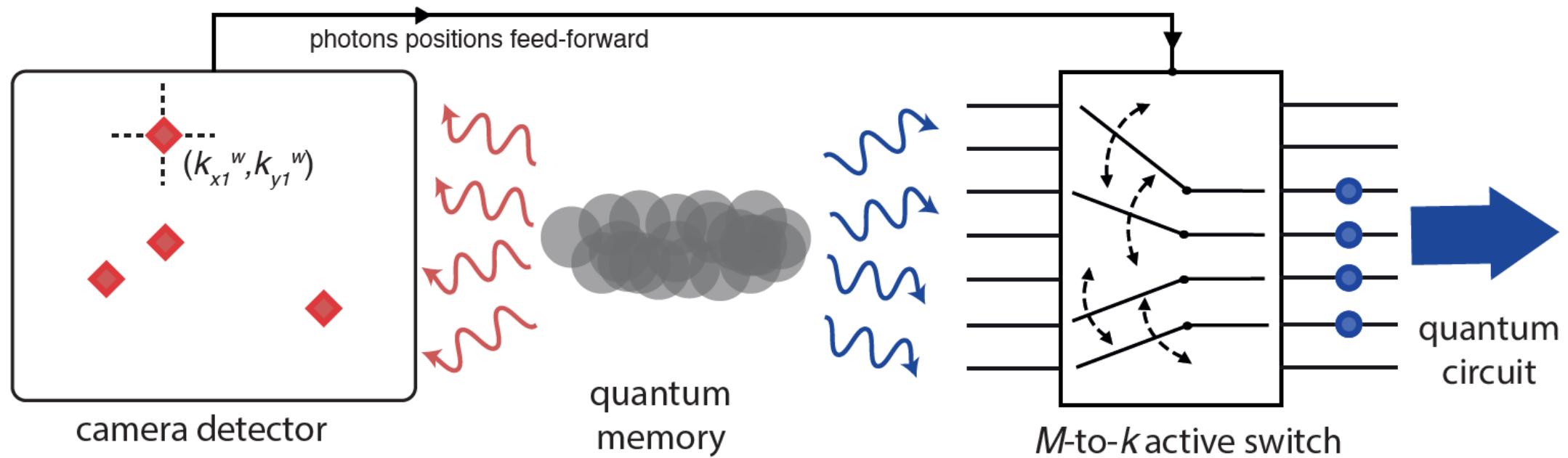


Wavevector Multiplexing

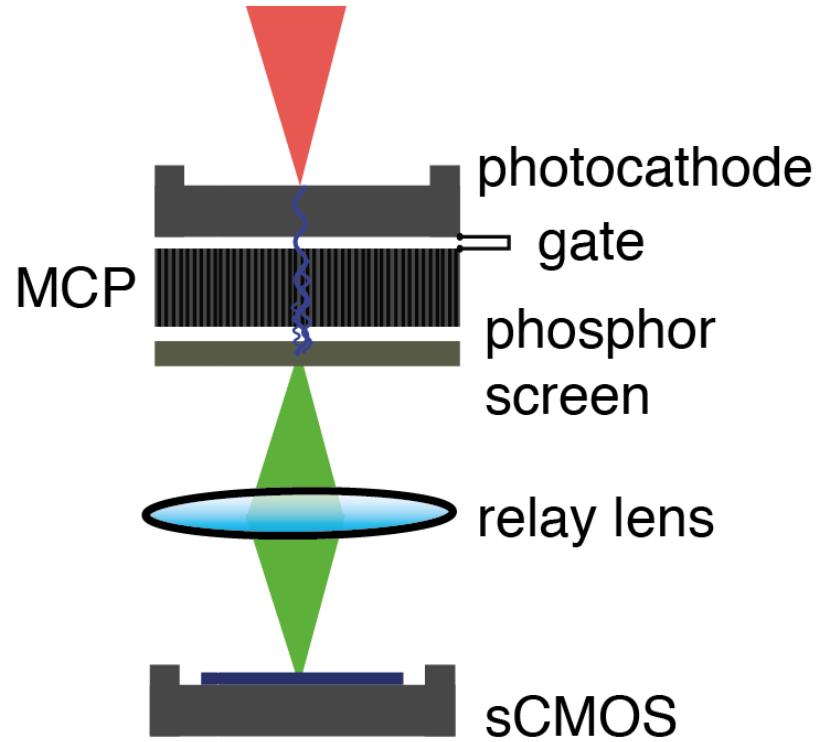


Radosław Chrapkiewicz, Michał Dąbrowski, and Wojciech Wasilewski
Phys. Rev. Lett. 118, 063603

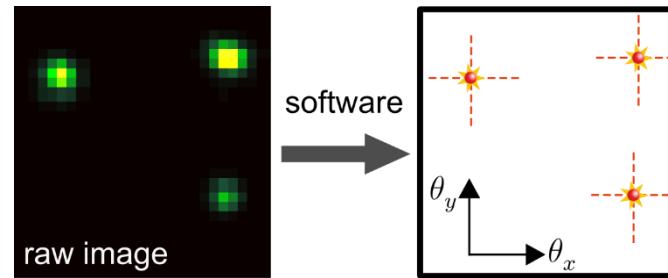
Deterministic single and multi-photons



I-sCMOS camera

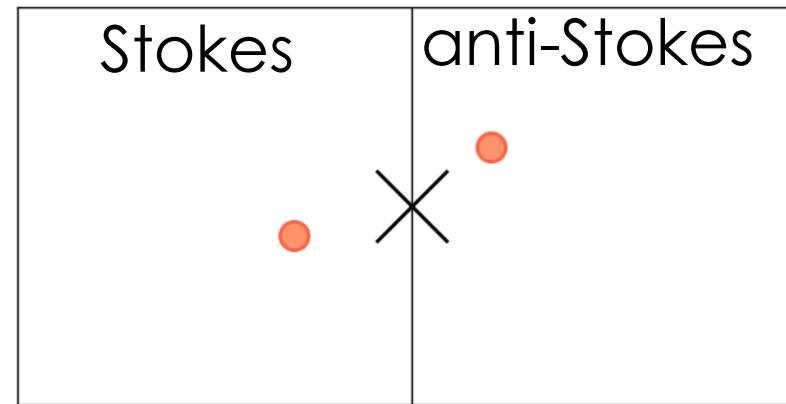


real-time image processing



Stokes

anti-Stokes



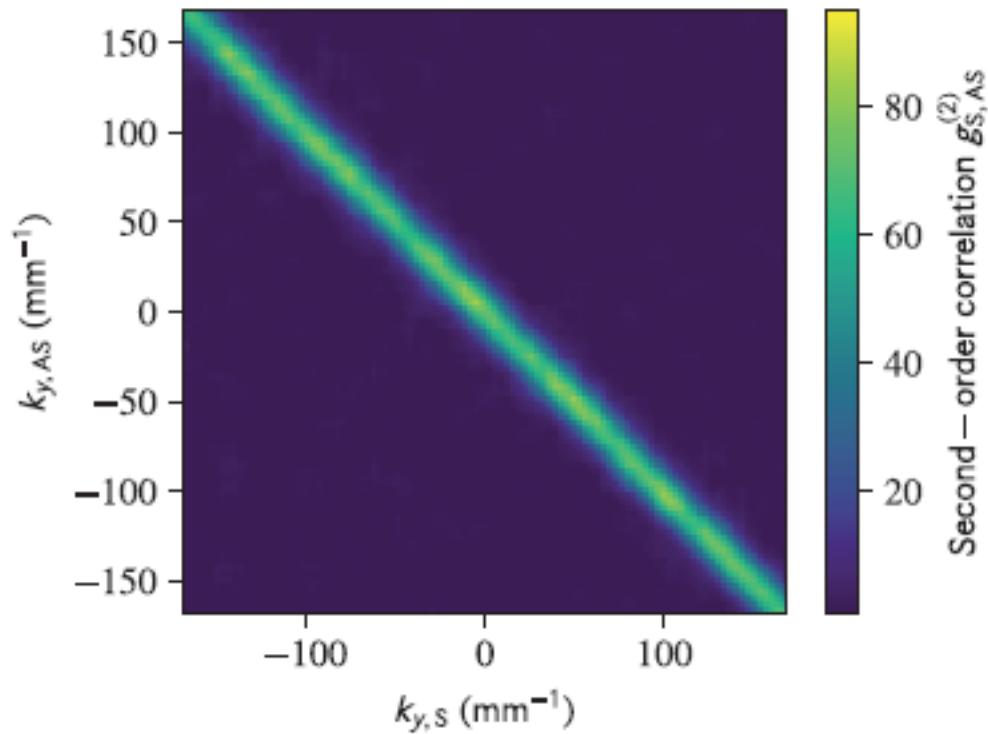
R. Chrapkiewicz, M. Jachura, K. Banaszek, W. Wasilewski, Nat. Photonics **10**, 576 (2016)

M. Jachura, R. Chrapkiewicz, W. Wasilewski, R. Demkowicz-Dobrzański, K. Banaszek, Nat. Commun. **7**, 11411 (2016)

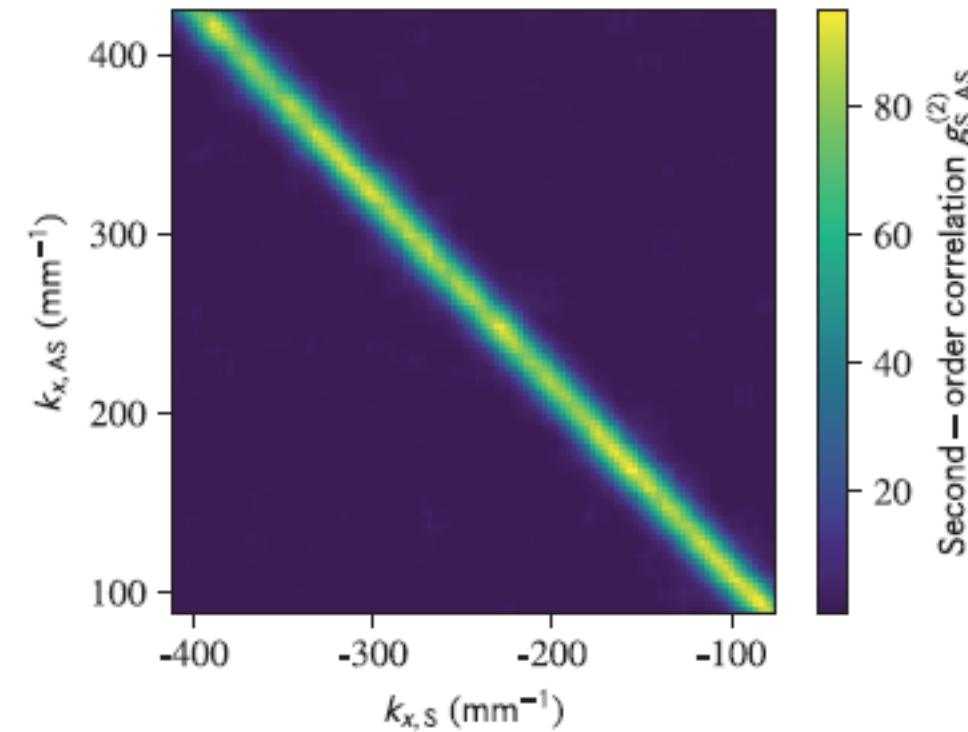
MP, M. Dąbrowski, M. Mazelanik, A. Leszczyński, M. Lipka, W. Wasilewski, Nat. Commun. **8**, 2140 (2017)

Photon number correlations

a

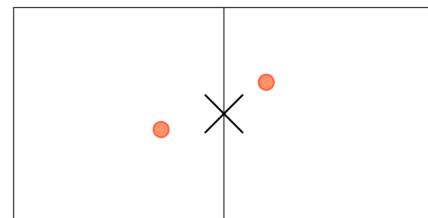


b



non-classical correlations

$$g^{(2)} = \frac{\langle n_S n_{AS} \rangle}{\langle n_S \rangle \langle n_{AS} \rangle} = 72 \pm 5 \gg 2$$



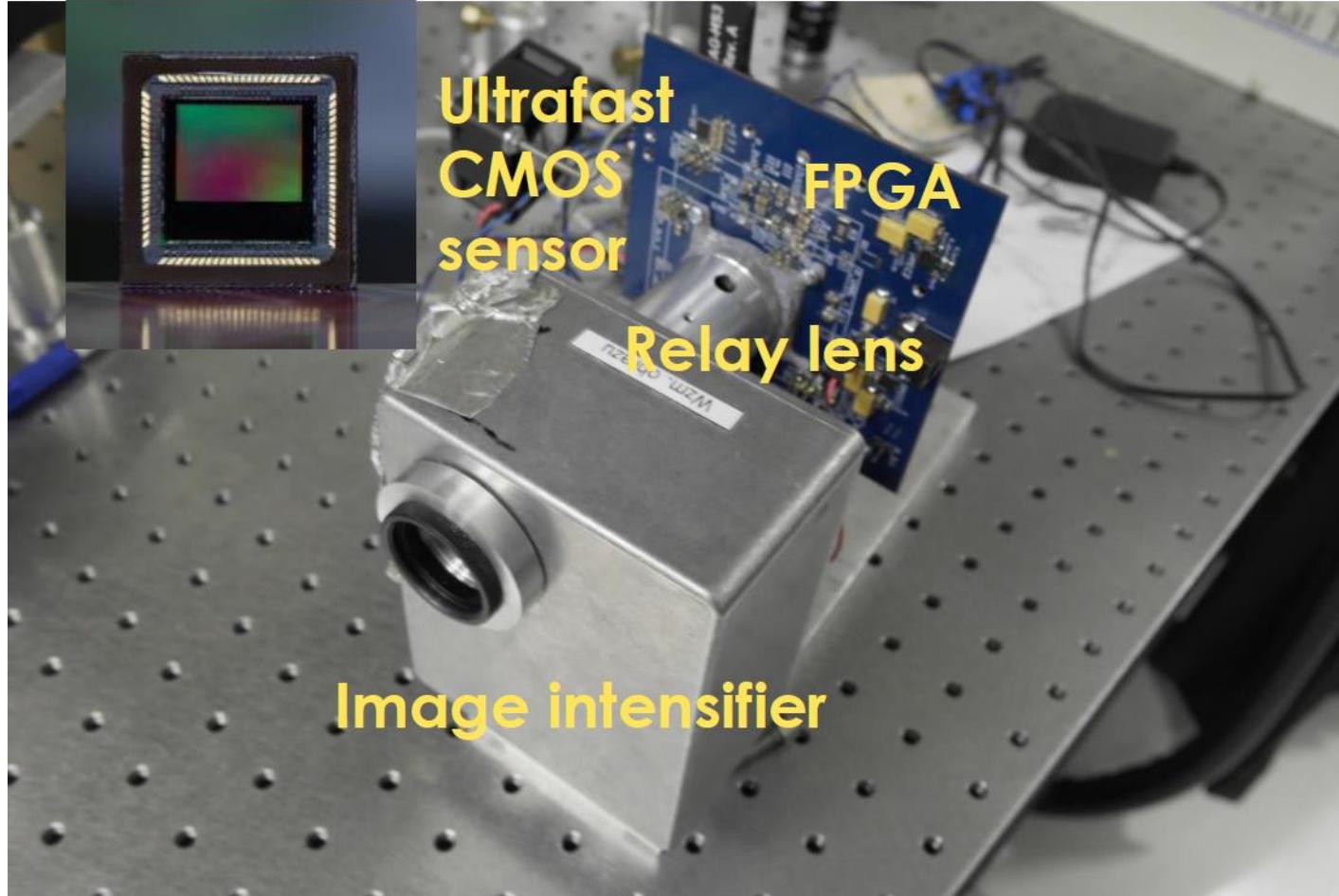
- >500 modes
- >50 μs storage

New system

Custom FPGA data processing

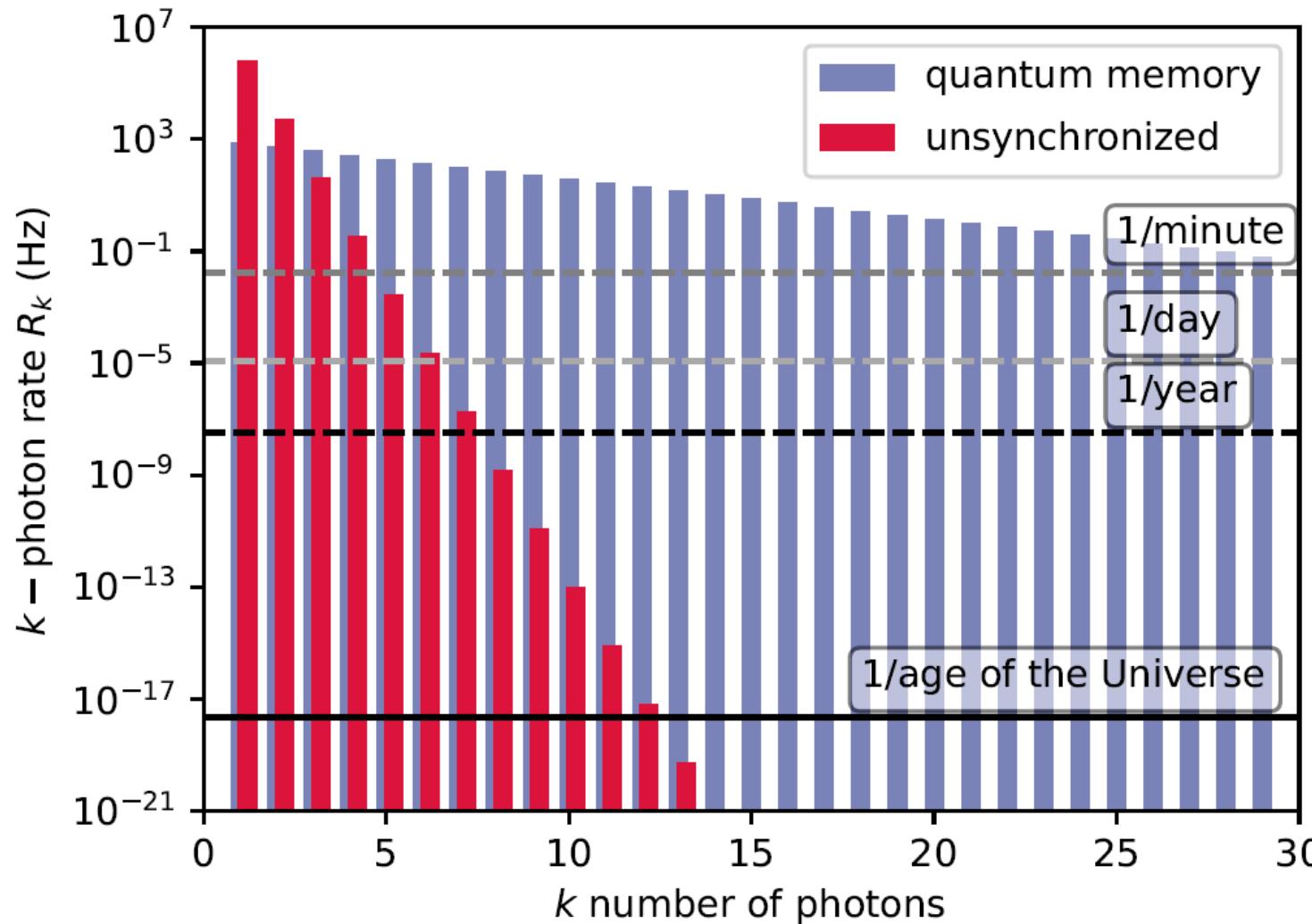
New custom high-voltage gating module

Now 100.000 frames per second, **~10 microseconds from detection to information**



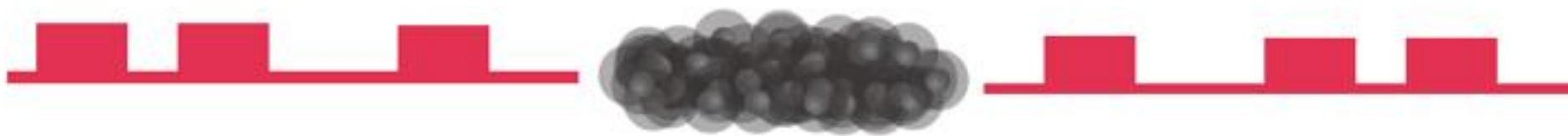
Optics Letters 46, 3009-3012 (2021)

Photon rate gains

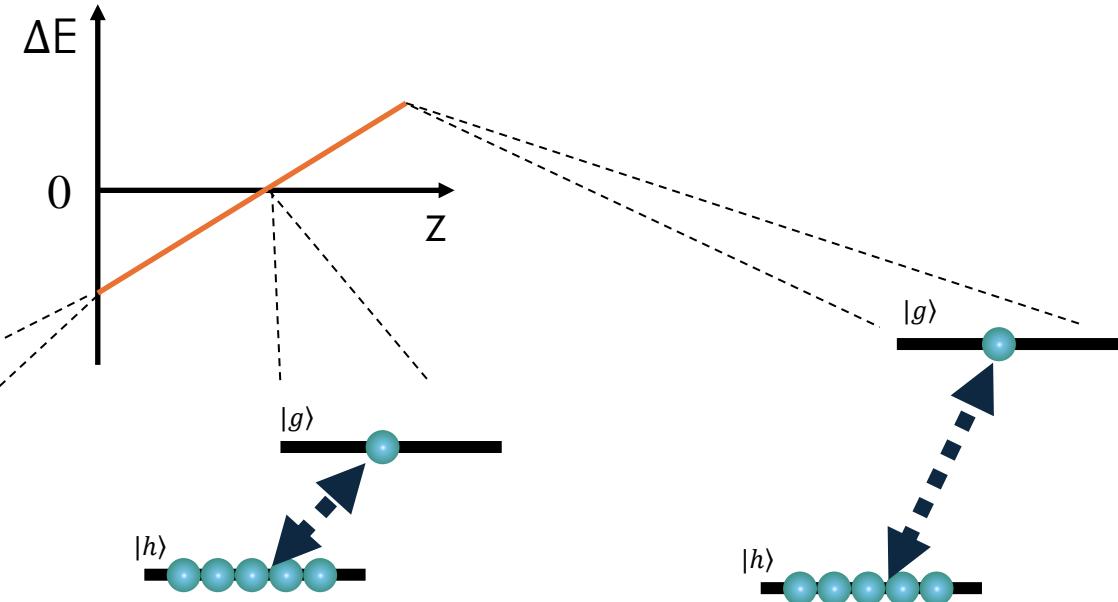
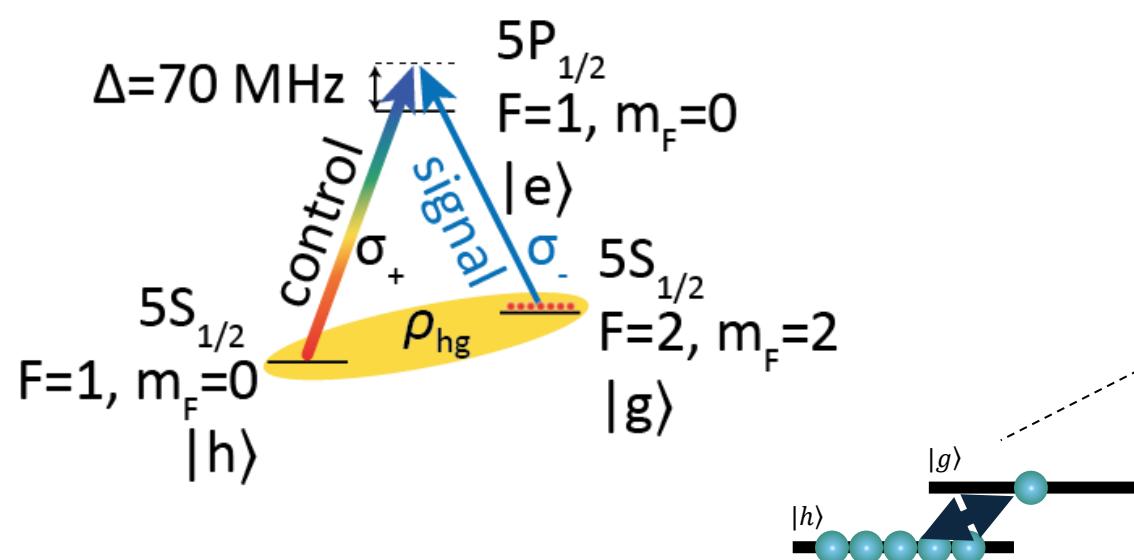


**1 kHz rep. rate
quantum memory**
vs
80 MHz rep. rate SPDC

Temporal multiplexing

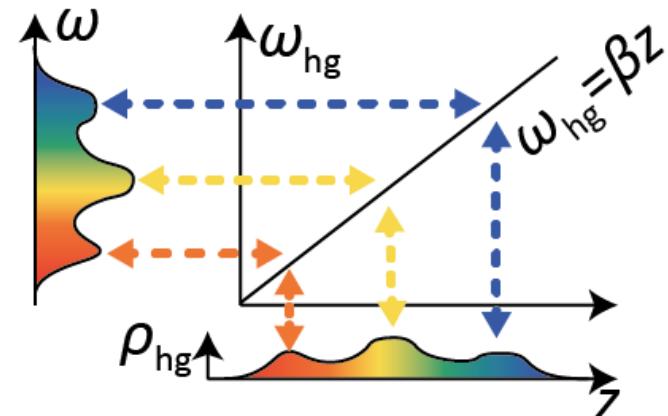


Gradient echo memory (GEM)



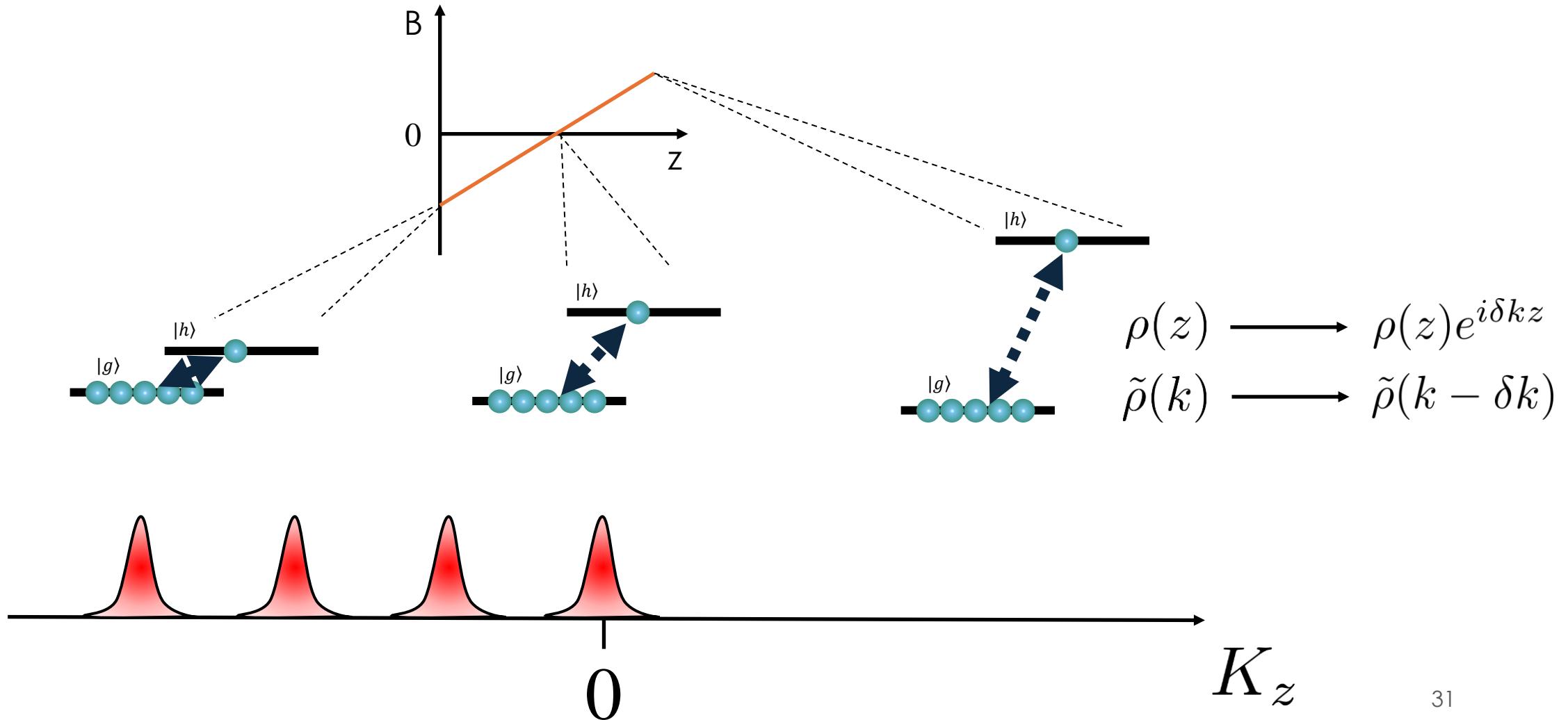
$$\frac{\partial \check{\rho}_{hg}(z, t)}{\partial t} = \frac{i}{\hbar} \frac{\Omega^*(t) dA(z, t)}{4\Delta - 2i\Gamma} - \frac{1}{2\tau} \check{\rho}_{hg}(z, t) + i\delta_{\text{tot}}(z, t) \check{\rho}_{hg}(z, t),$$

$$\frac{\partial A(z, t)}{\partial z} = -i \frac{\hbar \Omega(t) \check{\rho}_{hg}(z, t)/d + A(z, t) \Gamma}{2\Delta + i\Gamma} \frac{1}{2} g n(z),$$

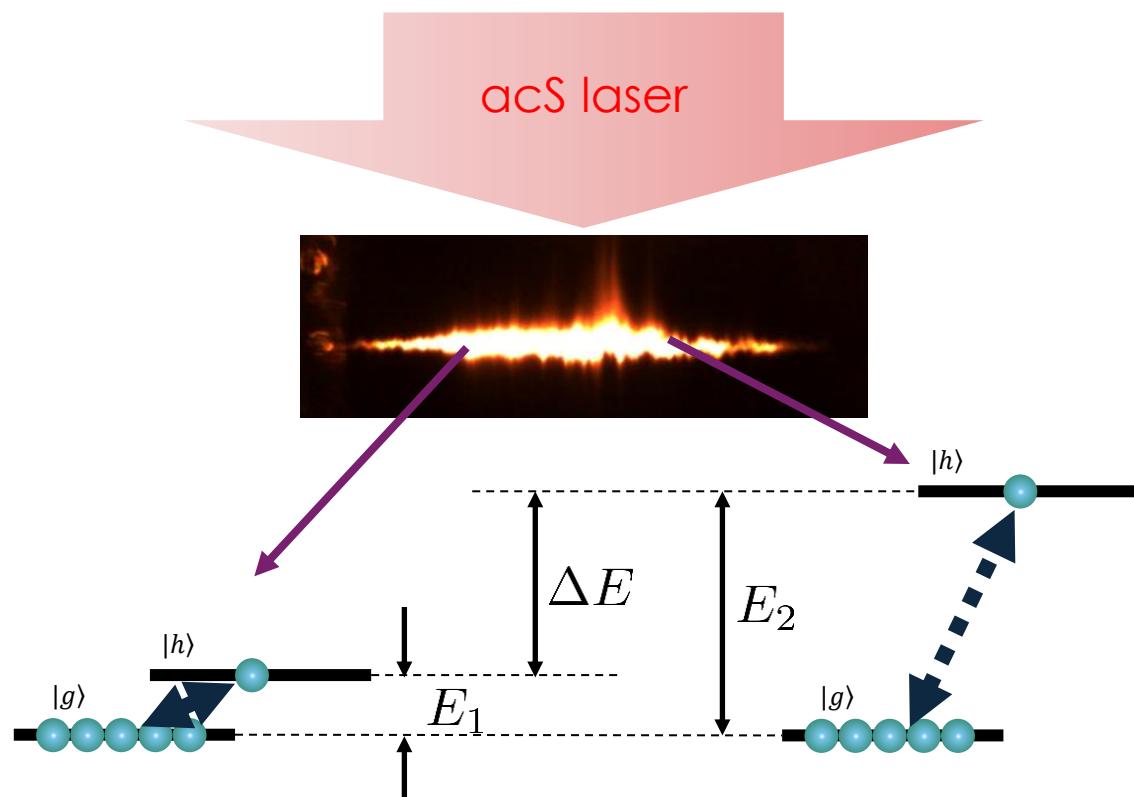


Hosseini et al., Nature 461, 241 (2009)

Spin-wave phase modulation (GEM)

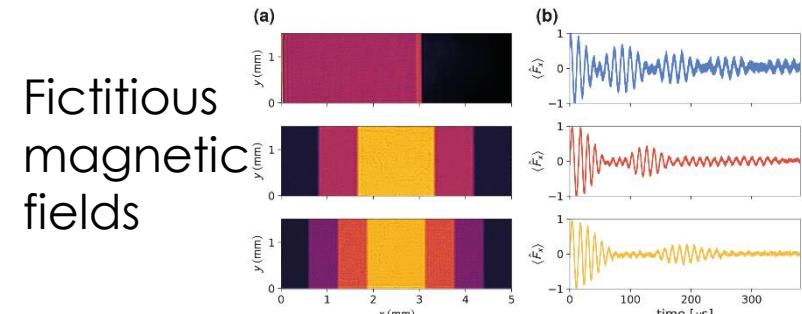


ac-Stark spin-wave phase modulation



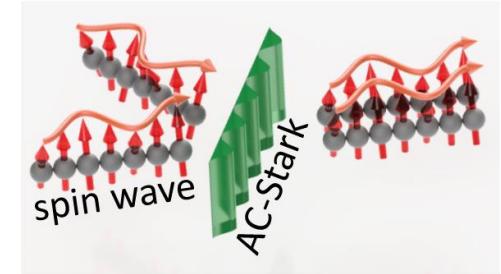
$$\Delta\varphi(y, z) = \Delta E(y, z)T/\hbar$$

Differential phase accumulated during free evolution



Opt. Lett. 43, 1147 (2018)

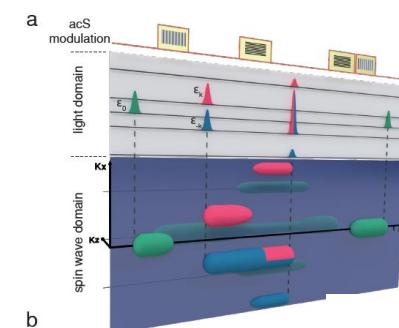
Spin-wave
Hong-Ou-
Mandel
interference



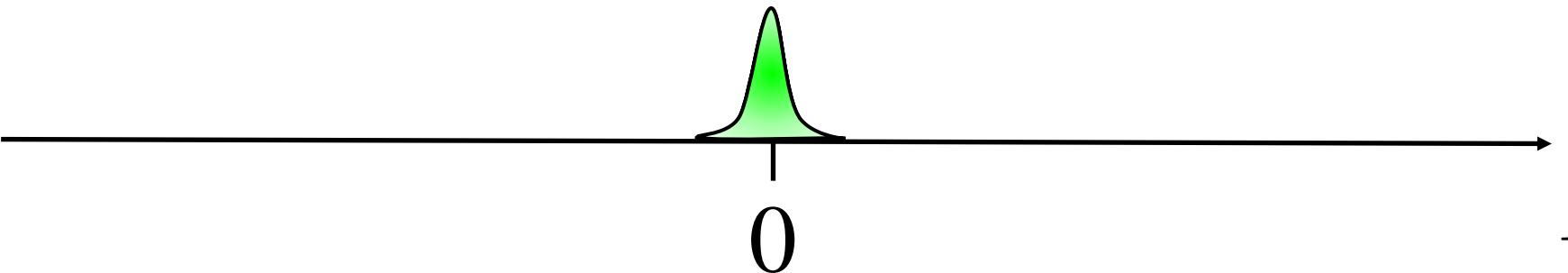
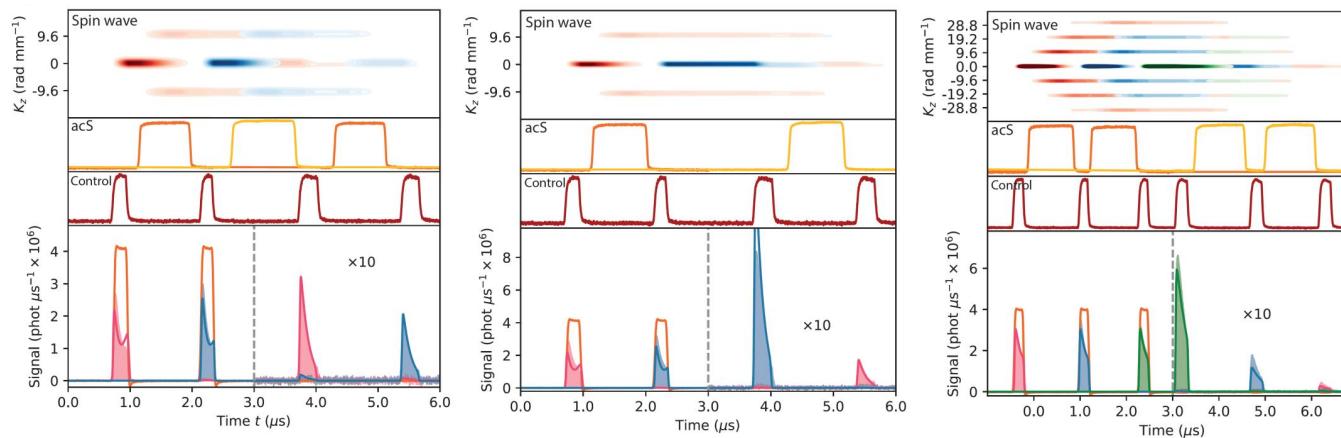
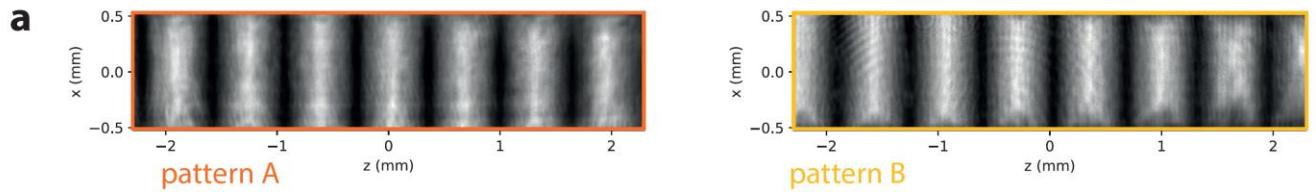
Phys. Rev. Lett. 122, 063604 (2019)

Spin-wave
processor of
stored optical
pulses

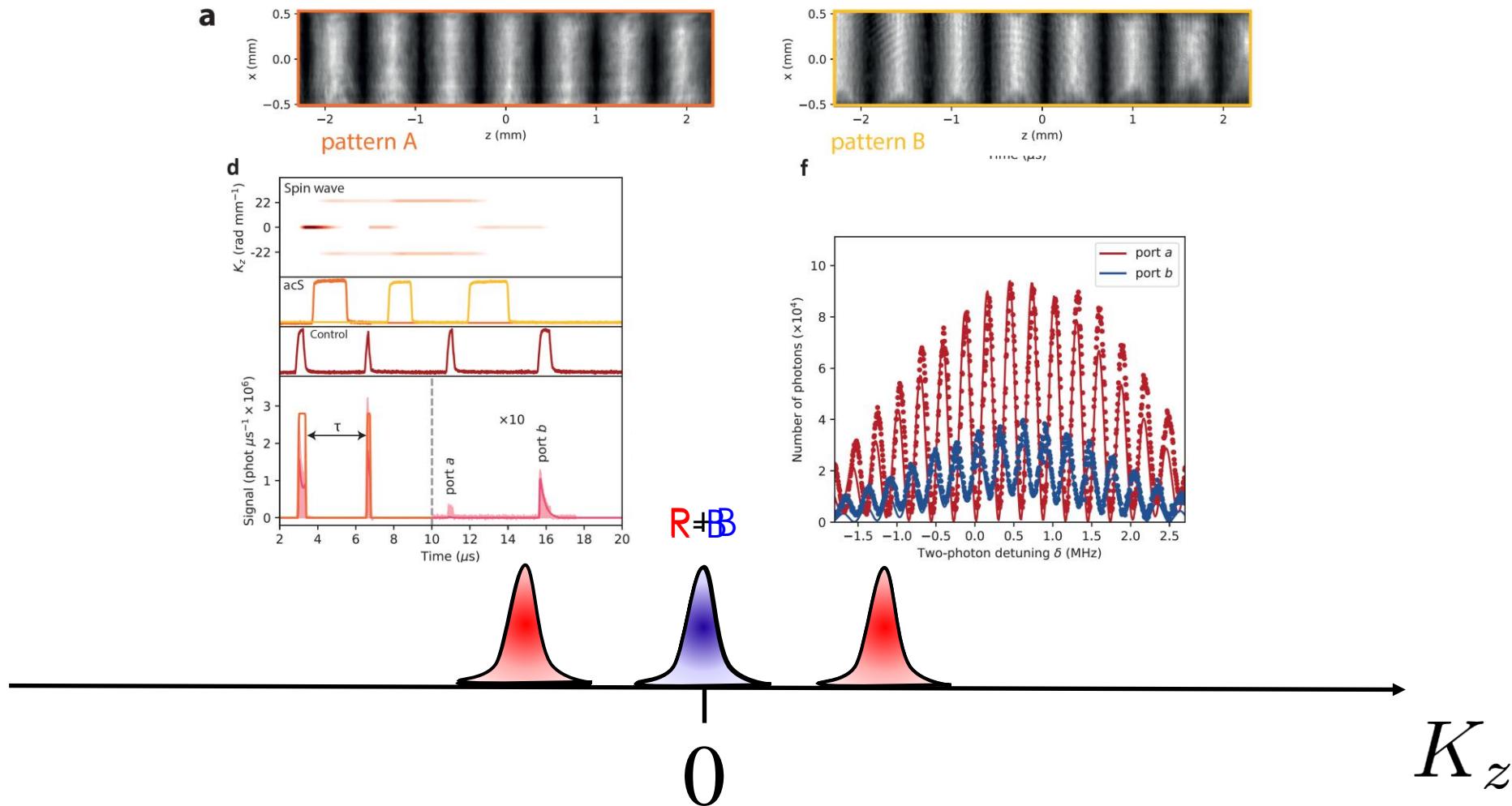
npj Quantum Information 5, 22 (2019)



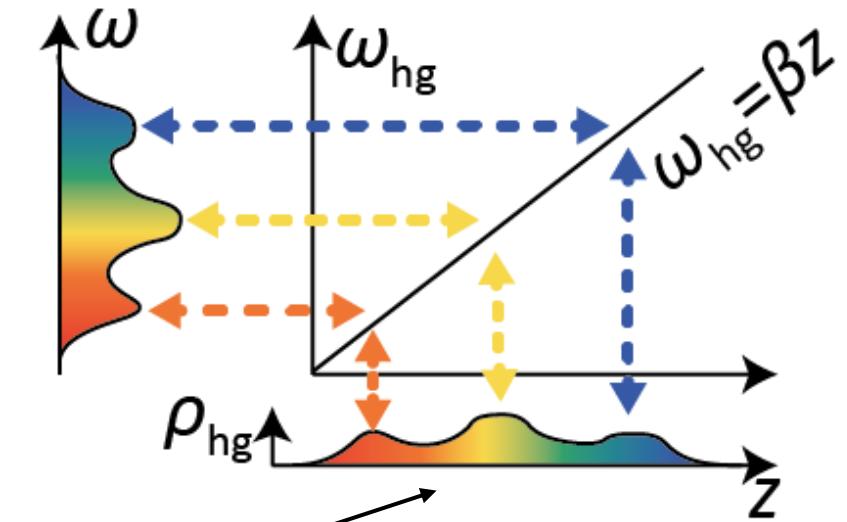
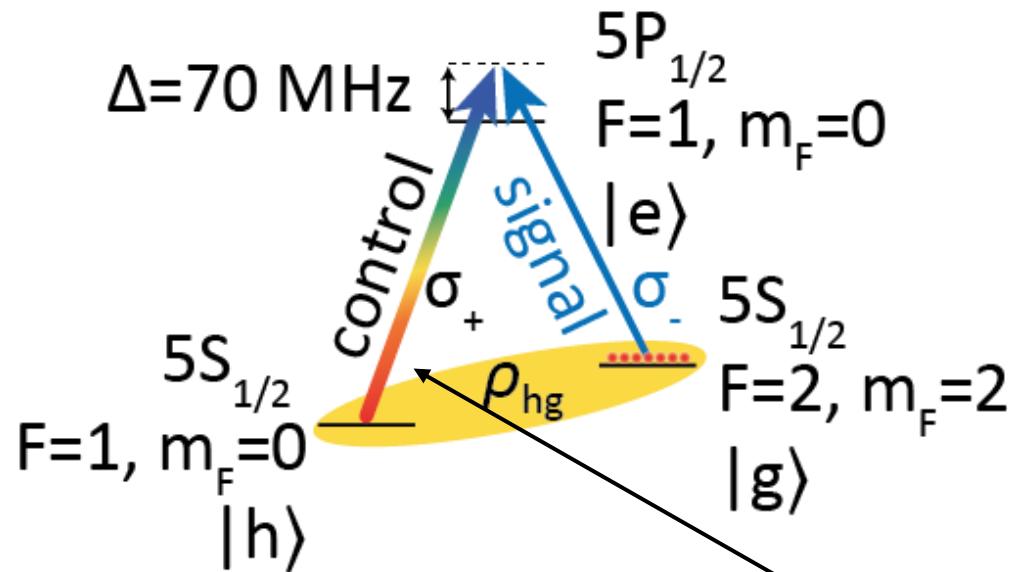
ac-Stark GEM



Spin-wave splitter



Time-lens and spectro-spatial mapping



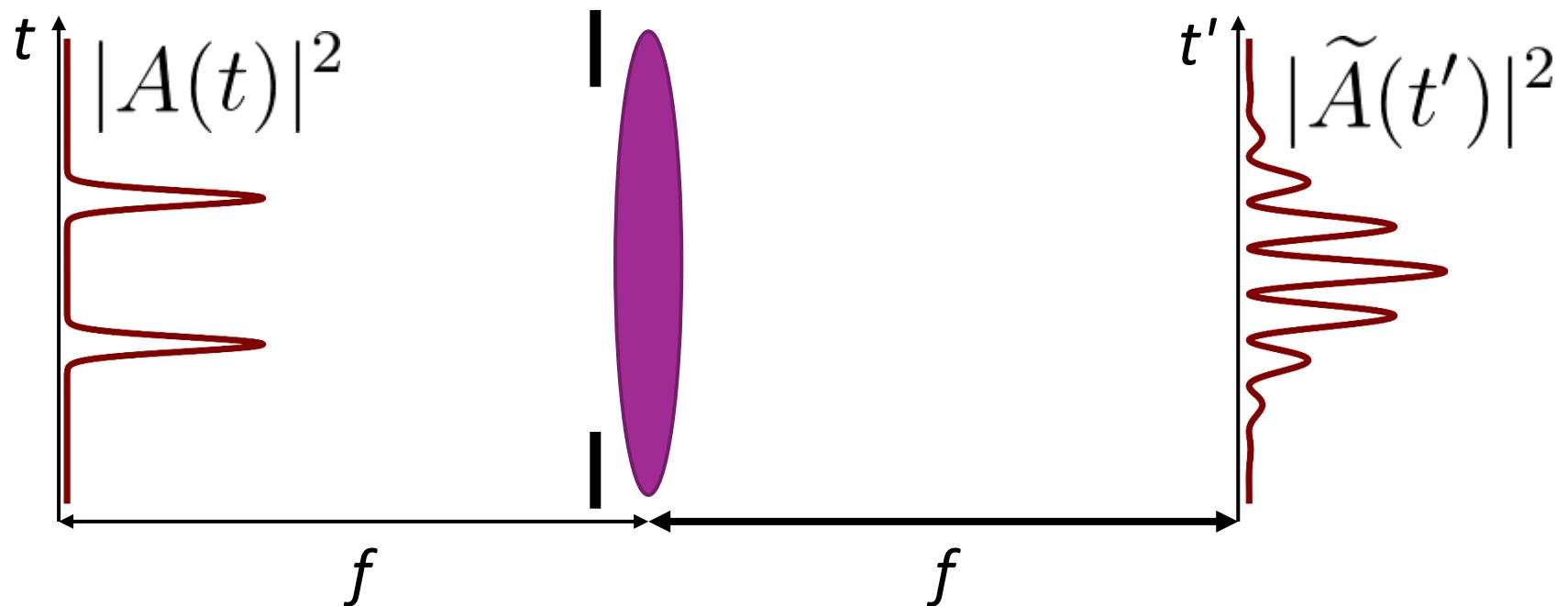
$$\rho_{hg}(z) \propto \tilde{A}(\beta z)$$

Time-lens realized
by chirped control field

$$\delta\omega(t) = \alpha t$$

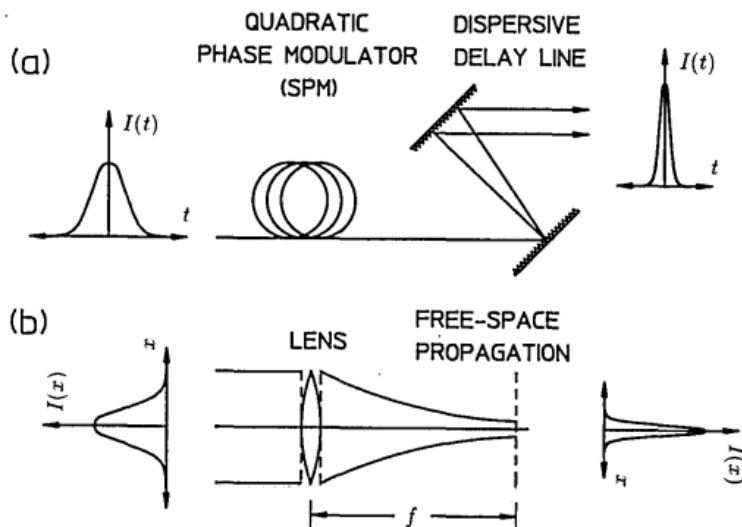
$$\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_t} & 1 \end{bmatrix}$$

Far-field temporal imaging



$$|\tilde{A}(t')|^2 = |\mathcal{F}_t[A(t)](t')|^2$$

Temporal imaging

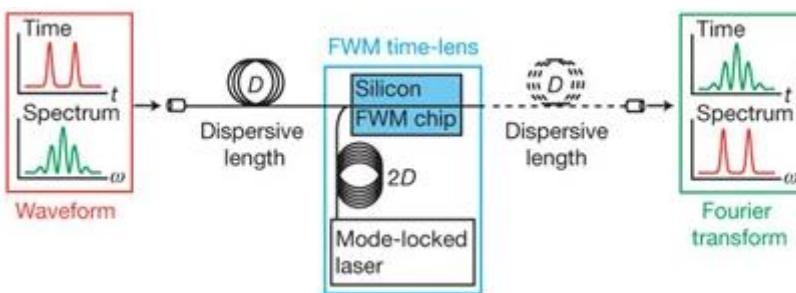


Opt. Lett. 14, 630 (1989)

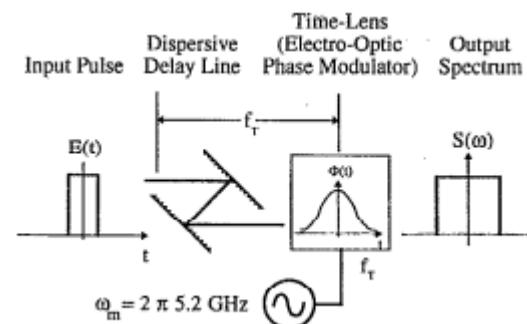
- Spectral conversion
- Bandwidth manipulation
- Temporal ghost imaging
- Characterization of the time-frequency entanglement
- Manipulation of field-orthogonal temporal modes

Existing solutions are compatible with solid-state emission
(high bandwidth, low spectral resolution)

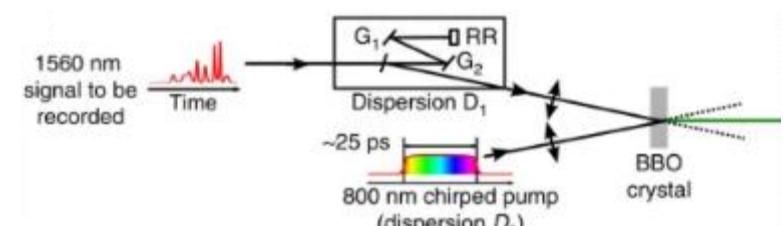
No solution for narrowband atomic emission



Nature 456, 81–84 (2008)

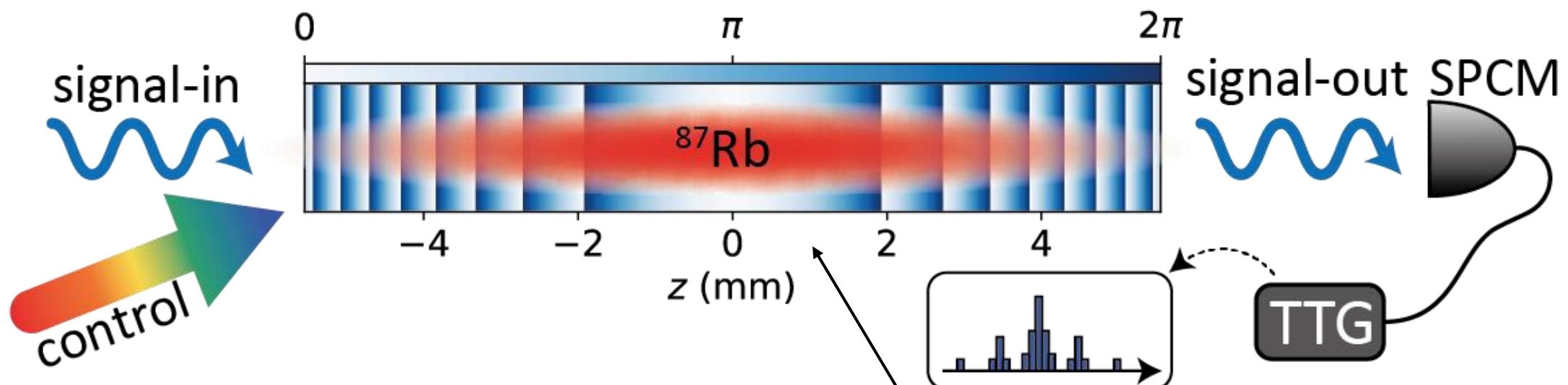


Appl. Phys. Lett. 64, 270–272 (1994)



Nat. Commun. 7, 13136 (2016)

Temporal propagation



Temporal „free-space” propagation
– a quadratic phase in frequency

$$\tilde{A}(\omega) = \mathcal{F}_t[A(t)](\omega)$$

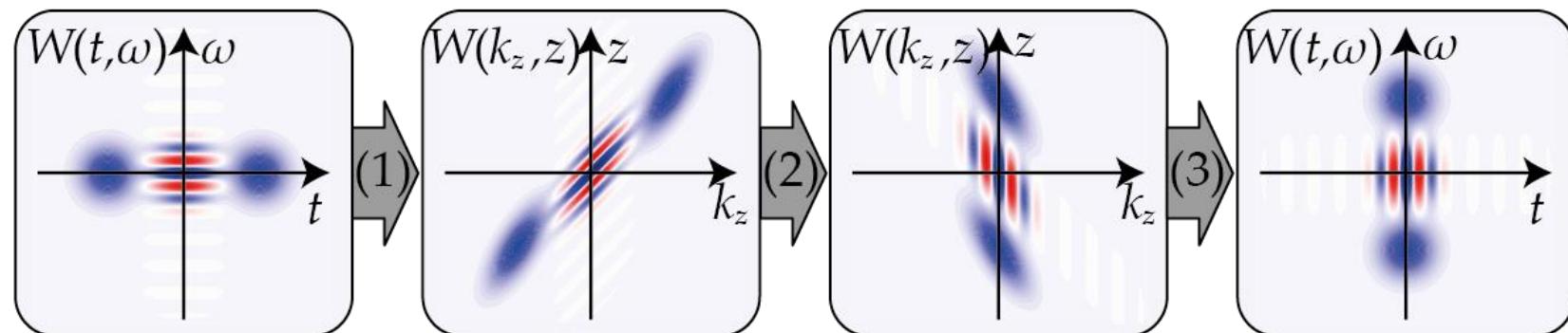
$$\tilde{A}(\omega) \rightarrow \tilde{A}(\omega) \exp[-i(f_t/\omega_0)\omega^2]$$

Thanks to spectro-spatial mapping the temporal propagation is realized by imposing a quadratic phase (Fresnel) profile onto the atomic coherence ρ_{hg}

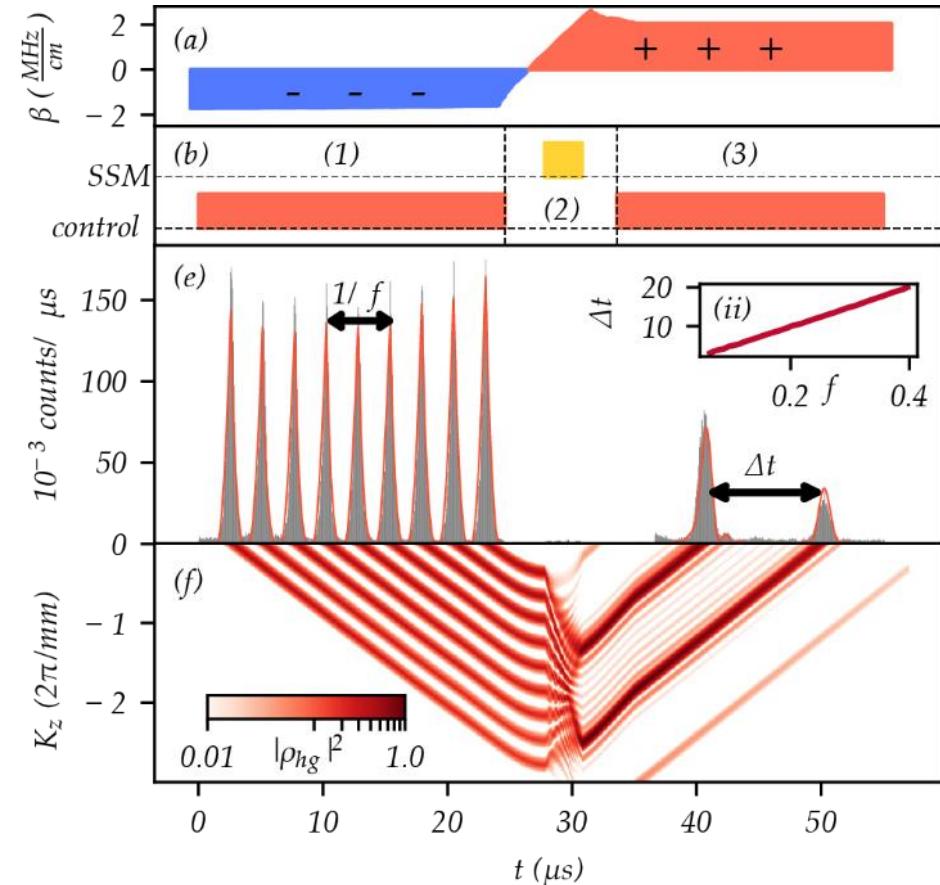
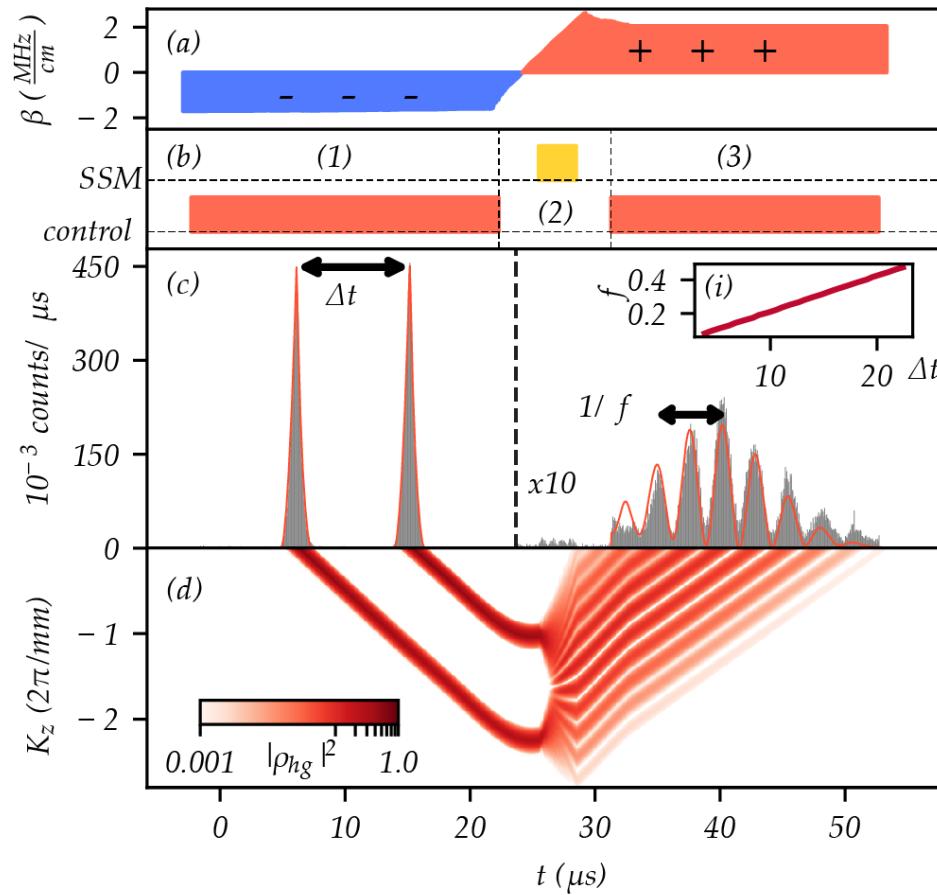
FF-TI (QMTI) - Rotating Wigner function

$$W(t, \omega) = 1/\sqrt{2\pi} \int_{-\infty}^{\infty} A(t + \xi/2) A^*(t - \xi/2) \exp(-i\omega\xi)$$

$$\begin{bmatrix} t' \\ \frac{\omega'}{\omega_0} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_t} & 1 \end{bmatrix} \begin{bmatrix} 1 & f_t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_t} & 1 \end{bmatrix} \begin{bmatrix} t \\ \frac{\omega}{\omega_0} \end{bmatrix} = \begin{bmatrix} 0 & f_t \\ -\frac{1}{f_t} & 0 \end{bmatrix} \begin{bmatrix} t \\ \frac{\omega}{\omega_0} \end{bmatrix}$$

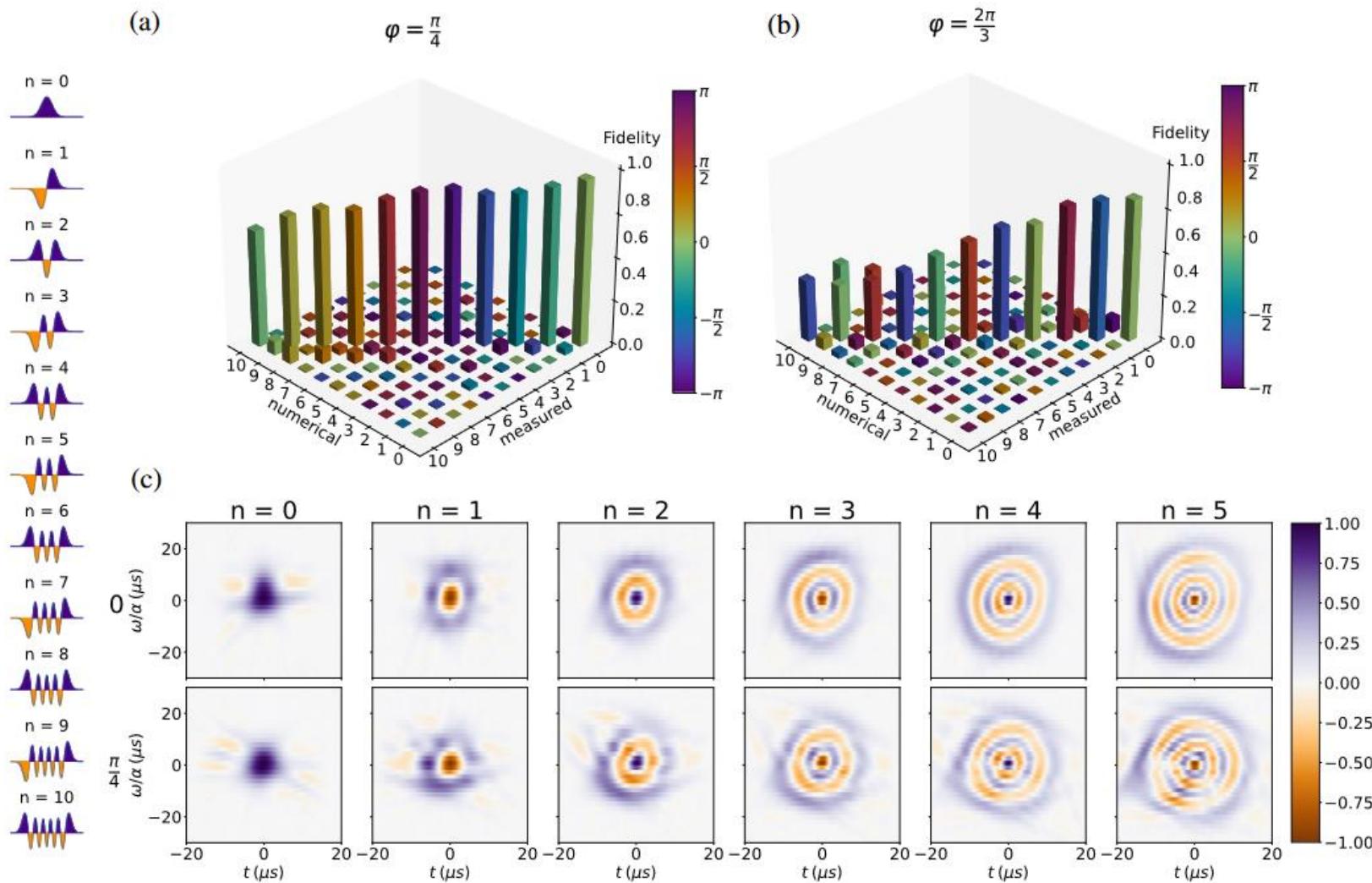


Example input/output

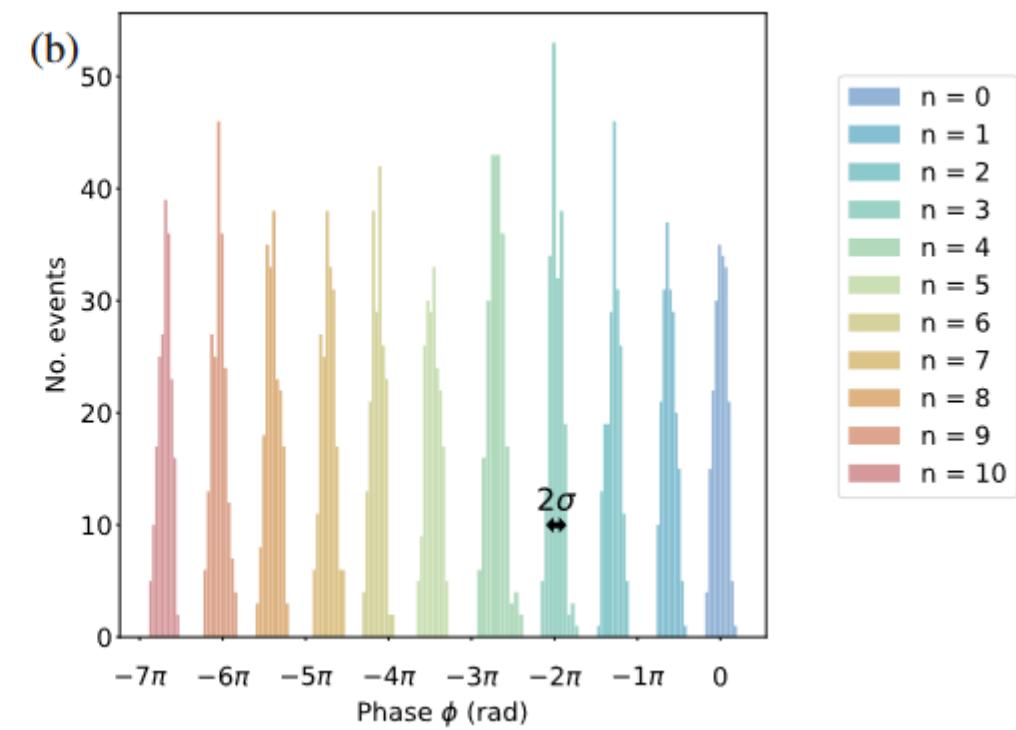
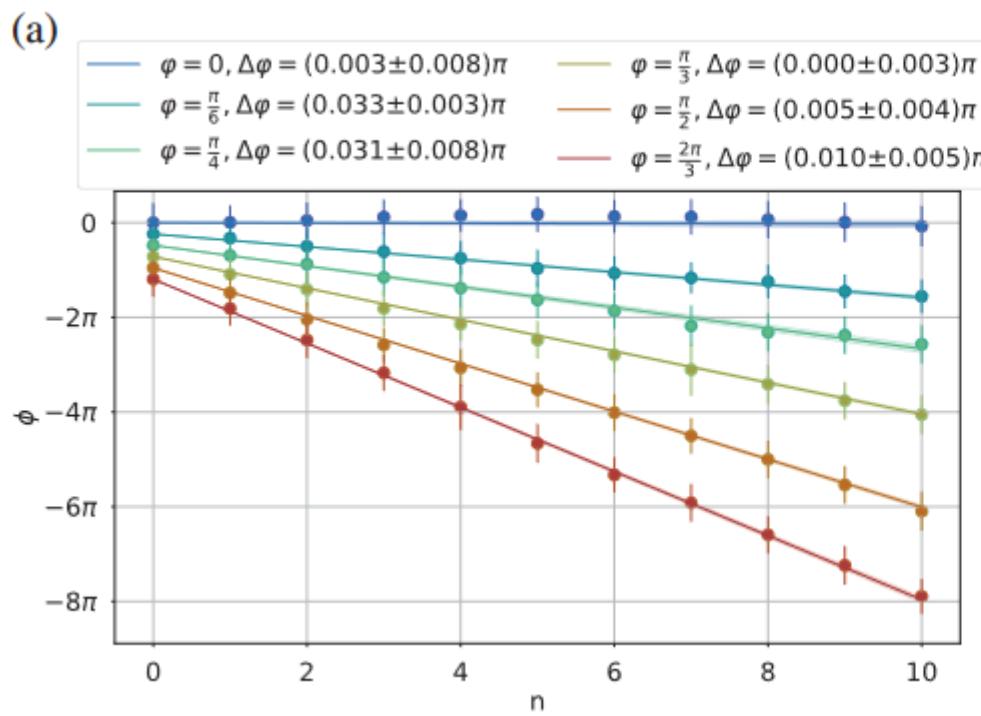


$$\left(\left(\tilde{A}(\alpha t) \exp(-i(\alpha/2)t^2) \right) * \zeta(\beta t) * \zeta(\beta t) \right) \exp(i(\alpha/2)t^2)$$

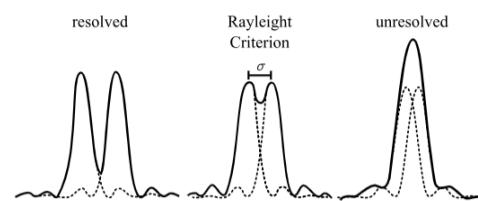
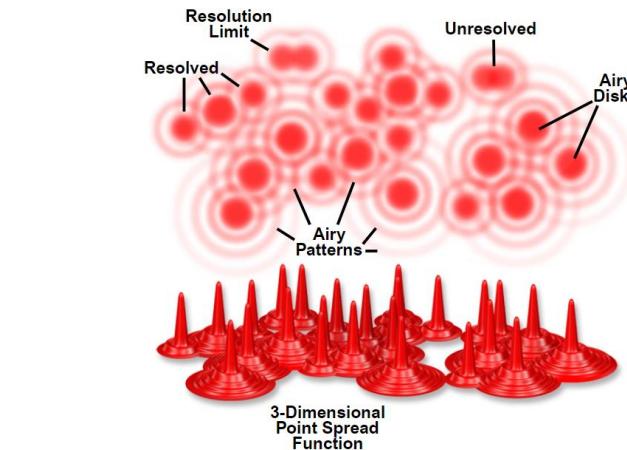
Rotation by arbitrary angle = the FrFT



Rotation by arbitrary angle = the FrFT



Imaging resolution - Rayleigh Criterion



$$\sigma = \frac{0.61\lambda}{\mu \sin \gamma} = \frac{0.61\lambda}{NA}$$

$$\theta = 1.2197 \frac{\lambda}{2R}$$



THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[FIFTH SERIES.]

OCTOBER 1879.

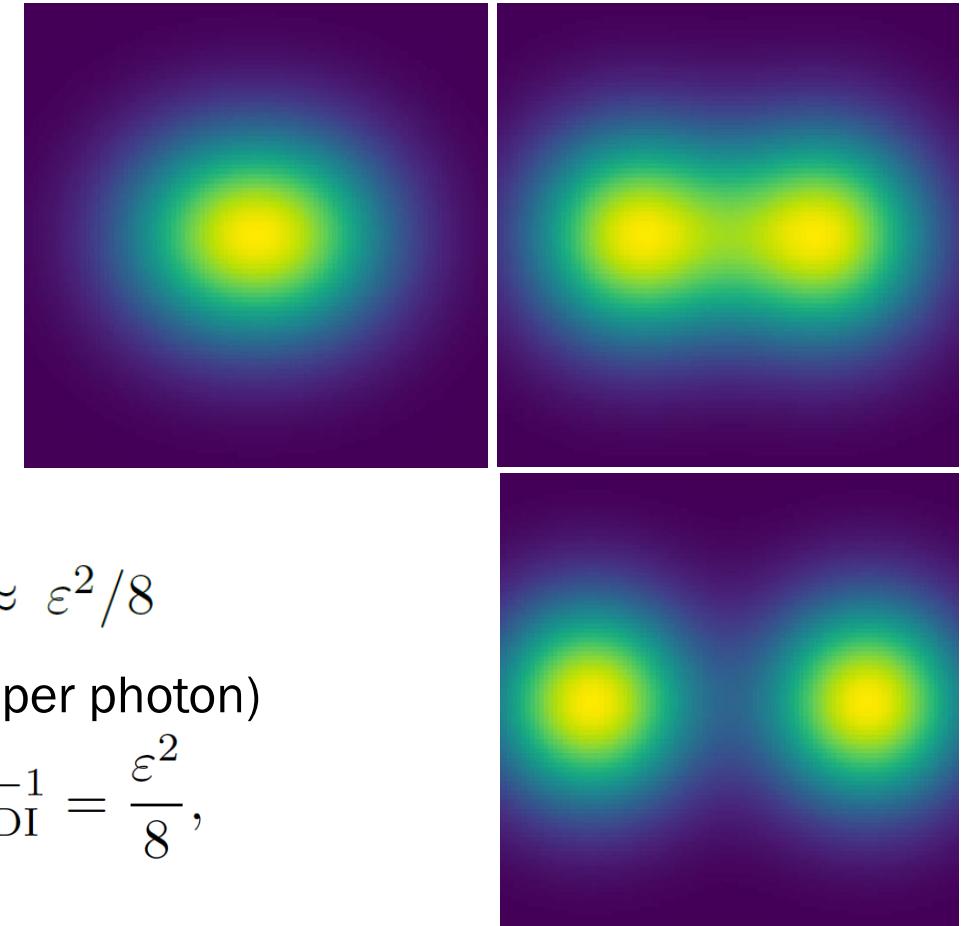
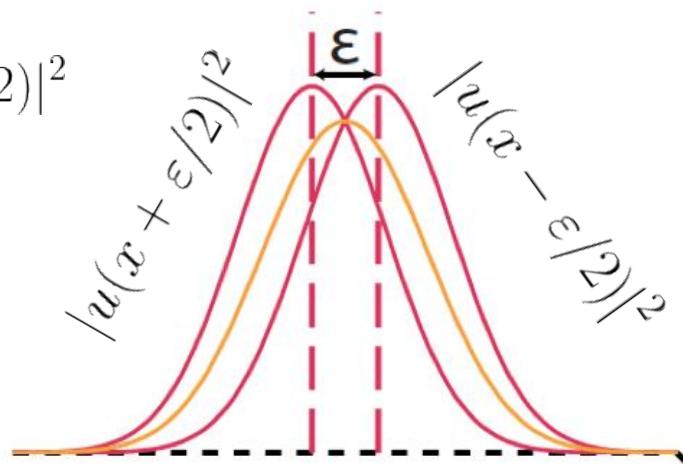
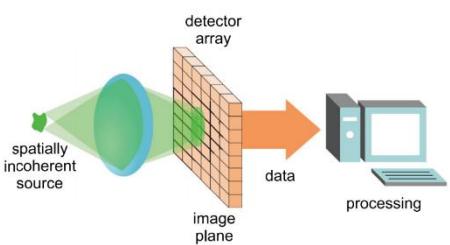
XXXI. *Investigations in Optics, with special reference to the Spectroscope.* By LORD RAYLEIGH, F.R.S.*
[Plate VII.]

§ 1. *Resolving, or Separating, Power of Optical Instruments.*

Rayleigh Limit

Two point sources:

$$|u(x + \varepsilon/2)|^2 + |u(x - \varepsilon/2)|^2$$



$$\mathcal{F}_{\text{DI}} \approx \varepsilon^2/8$$

Cramér–Rao bound (CRB)

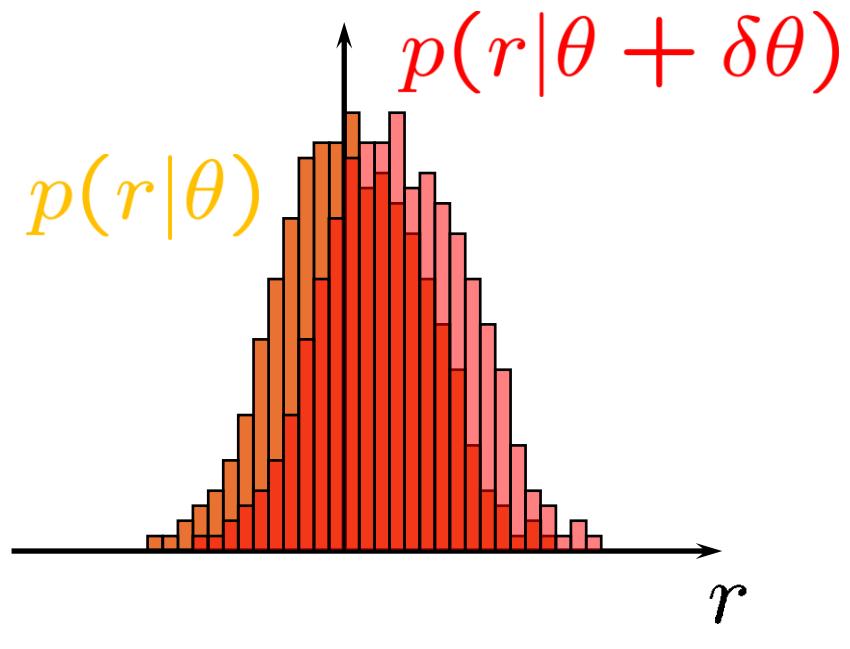
$$\Delta^2 \hat{\varepsilon} \geq \frac{1}{\mathcal{F}}, \quad \mathcal{F} = \int \frac{1}{p_\varepsilon(x)} \left(\frac{\partial}{\partial \varepsilon} p_\varepsilon(x) \right)^2 dx$$

Precision (per photon)

$$(\Delta^2 \varepsilon)_{\text{DI}}^{-1} = \frac{\varepsilon^2}{8},$$

Fisher information

$$F(\theta) = \sum_r p(r|\theta) \left(\frac{\partial}{\partial \theta} \log p(r|\theta) \right)^2$$

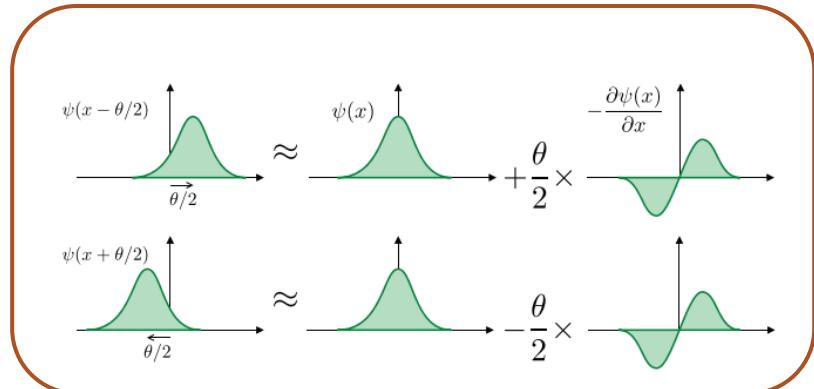


Cramér-Rao bound:
for unbiased estimators

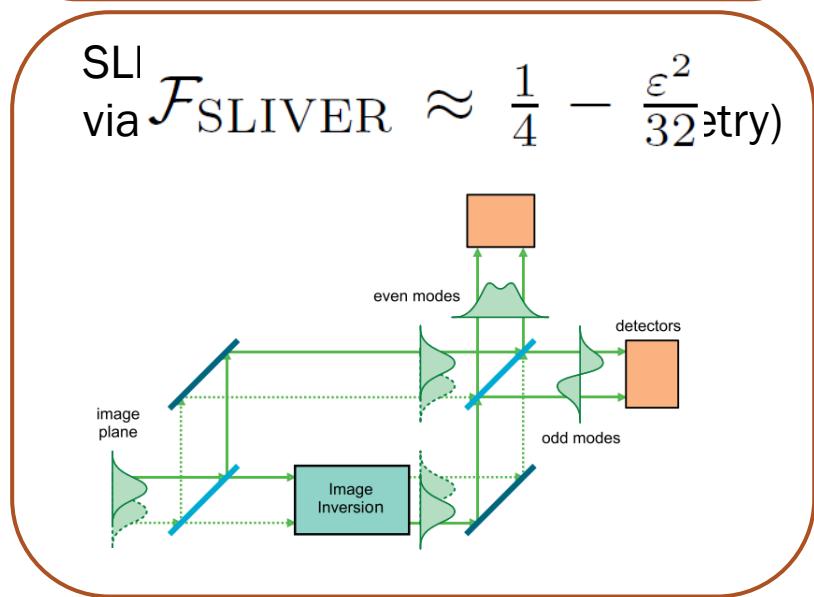
$$\Delta\theta \geq \frac{1}{\sqrt{N F(\theta)}}$$

$$(\Delta^2 \theta)^{-1} \leq F(\theta) N$$

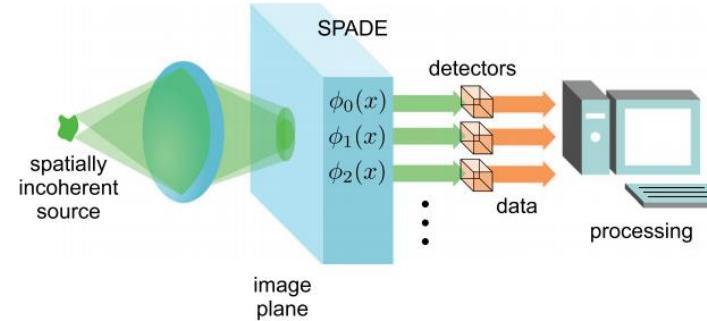
Beating the Rayleigh Limit more conventionally



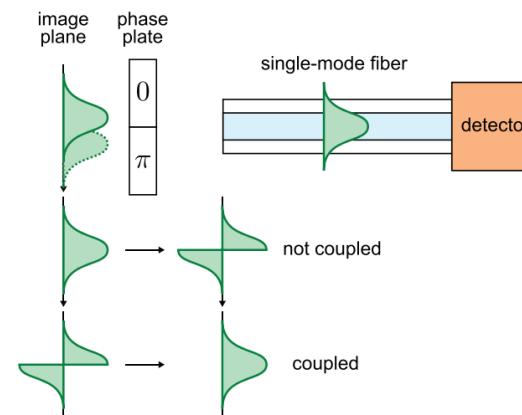
$$\text{SLI via } \mathcal{F}_{\text{SLIVER}} \approx \frac{1}{4} - \frac{\varepsilon^2}{32} \text{try})$$



SPADE (spatial-mode demultiplexing)



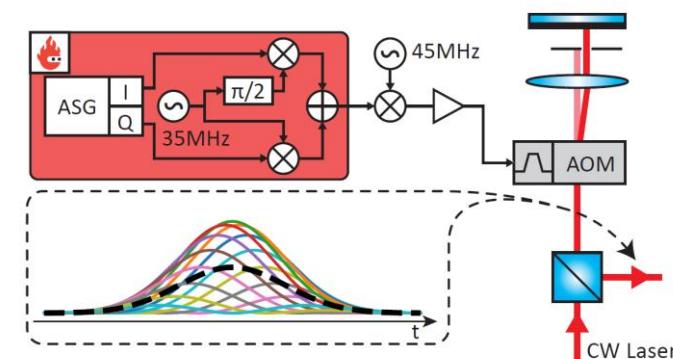
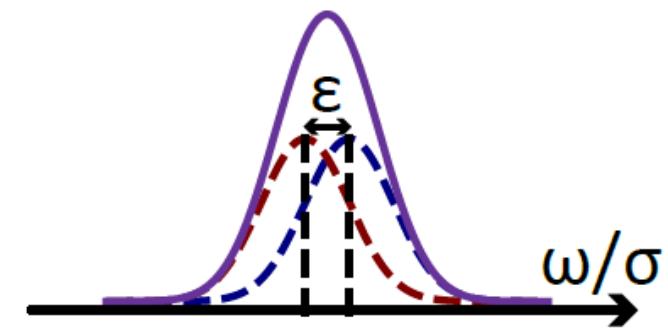
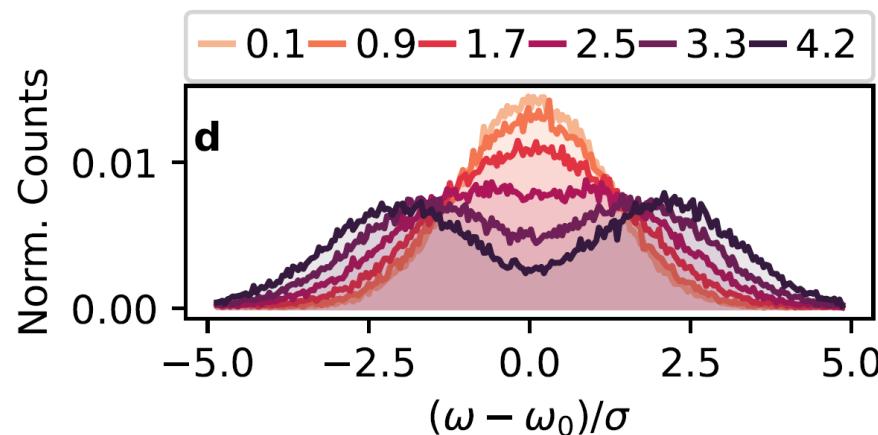
SPLICE (super-resolved position localization by inversion of coherence along an edge)



Two incoherent sources

$$\tilde{I}(\omega) = \frac{1}{2} \left(|\tilde{\psi}(\omega - \delta\omega/2)|^2 + |\tilde{\psi}_-(\omega + \delta\omega/2)|^2 \right)$$

$$\tilde{\psi}(\omega) = \tilde{\psi}_+(\omega) = \left(\sqrt{2\pi}\sigma \right)^{-1/2} \exp \left(-\frac{\omega^2}{4\sigma^2} \right)$$

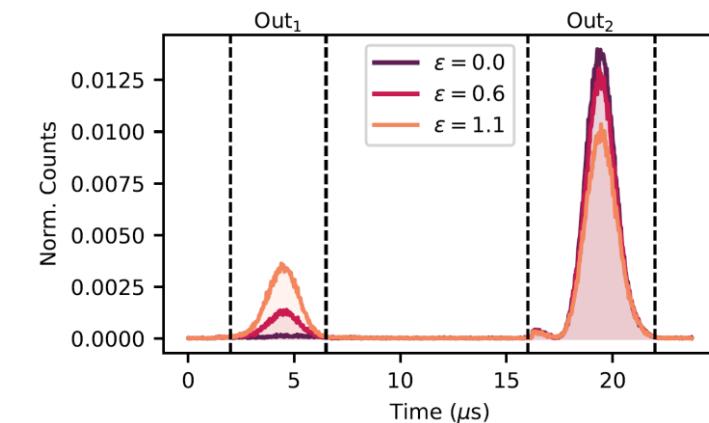
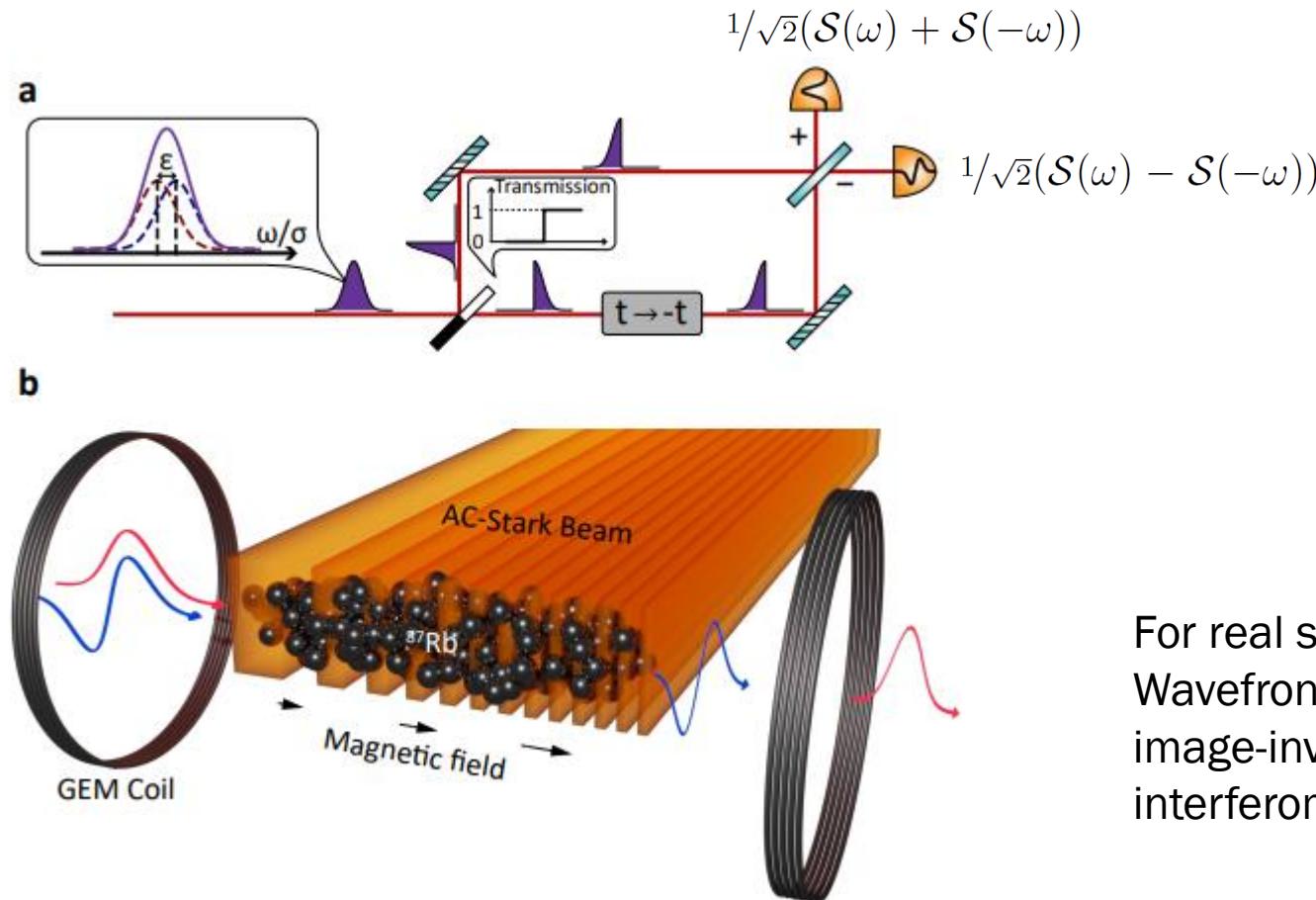


$$\mathcal{S}_\varphi(\omega) = \frac{1}{\sqrt{2}} \left(\tilde{\psi}_+(\omega - \sigma\varepsilon/2) + e^{i\varphi} \tilde{\psi}_+(\omega + \sigma\varepsilon/2) \right)$$

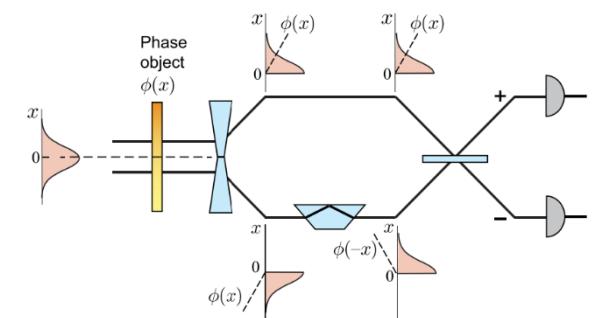
$$\mathcal{W}_S(t, \omega) = \tilde{\psi}_+^2(t) (\tilde{\psi}_+^2(\omega - \sigma\varepsilon/2) + \tilde{\psi}_+^2(\omega + \sigma\varepsilon/2))$$

PuDTAI

Pulse-division time-axis-inversion interferometer

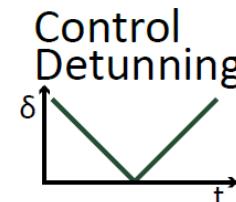
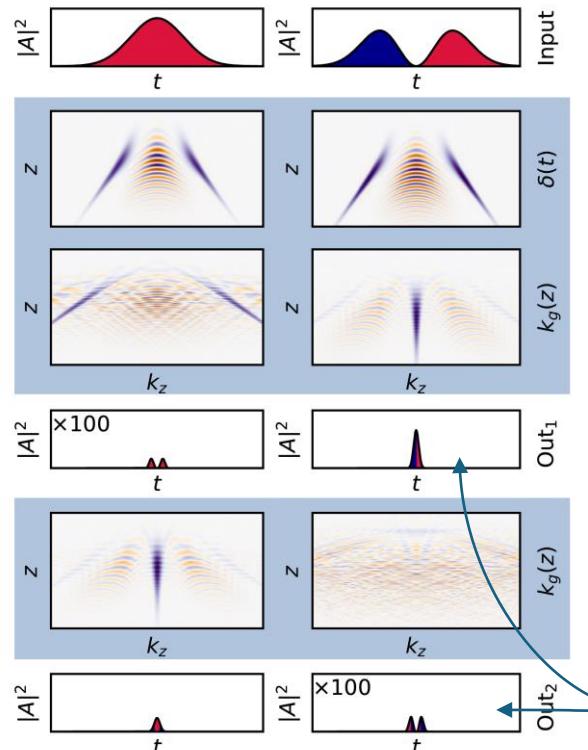


For real space imaging:
Wavefront-division
image-inversion
interferometer

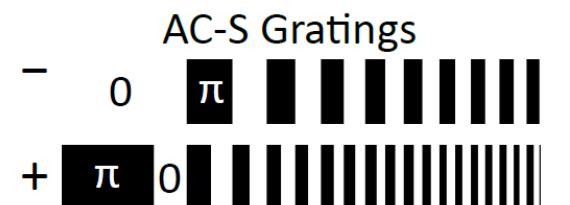


PuDTAI in phase space

$$\mathcal{W}(z, k_z) = \frac{1}{\sqrt{2\pi}} \int \varrho_{hg}(z + \xi/2) \varrho_{hg}^*(z - \xi/2) \exp(-ik_z\xi) d\xi$$



$$\delta(t) = \alpha|t|$$



$$k_g = \zeta z \quad k_g = 2\zeta z$$

The interference happens in the Fourier domain

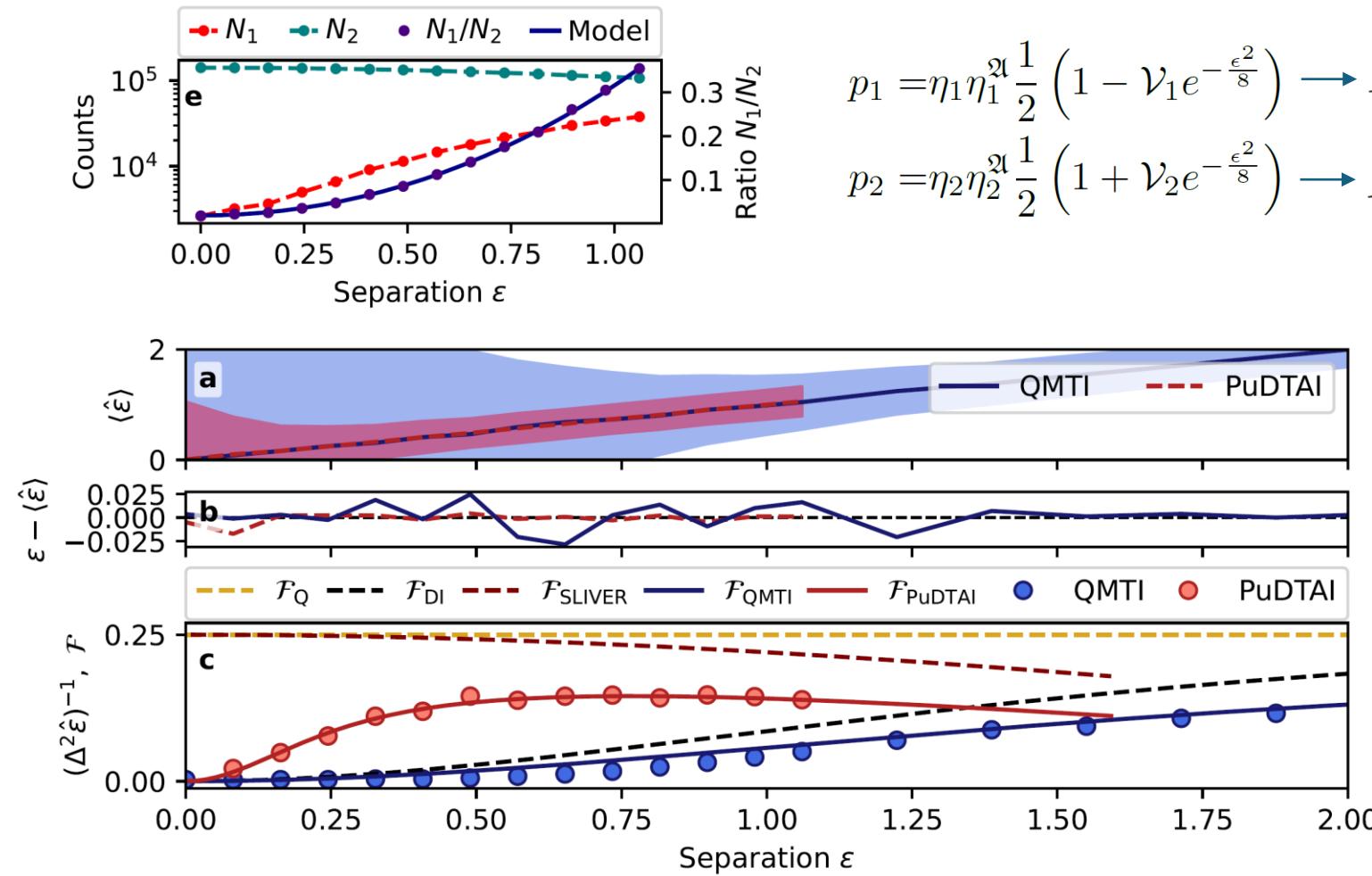
For the first half of the pulse:

$$\mathcal{A}_-(t) = \begin{cases} \mathcal{A}(t) & t < 0 \\ 0 & t \geq 0 \end{cases}$$

$$k_z \rightarrow k'_z = \zeta z$$

$$z \rightarrow z' = z - \frac{1}{\zeta} k_z$$

Separation estimation



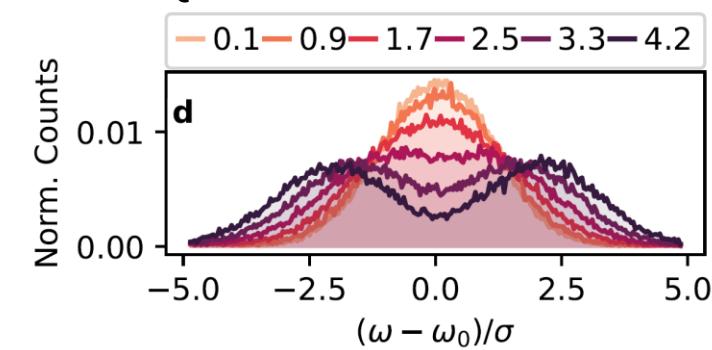
$$p_1 = \eta_1 \eta_1^2 \frac{1}{2} \left(1 - \mathcal{V}_1 e^{-\frac{\varepsilon^2}{8}} \right) \rightarrow N_1$$

$$p_2 = \eta_2 \eta_2^2 \frac{1}{2} \left(1 + \mathcal{V}_2 e^{-\frac{\varepsilon^2}{8}} \right) \rightarrow N_2$$

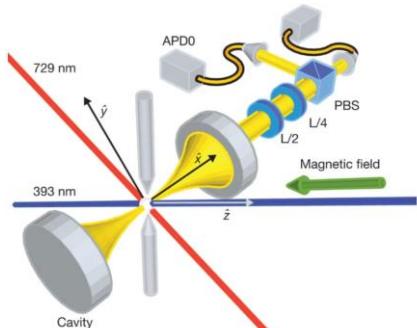
$$\hat{\varepsilon}(N_1/N_2)$$

Maximum likelihood estimation
in both cases

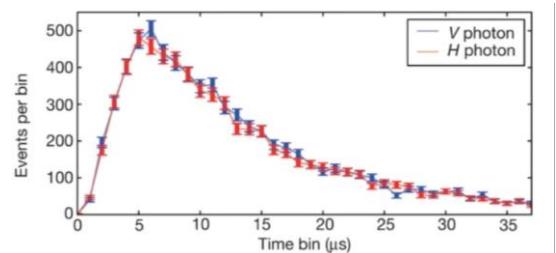
QMTI raw data:



Ultranarrowband optical spectroscopy

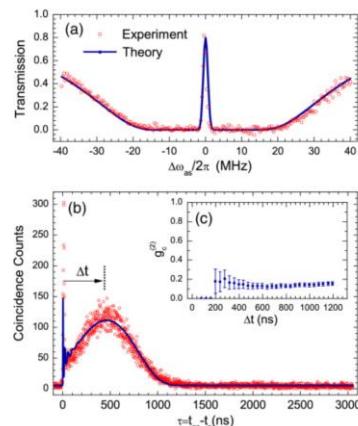
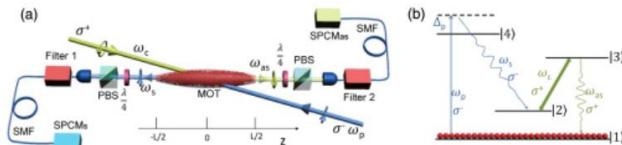


Single ions <100kHz



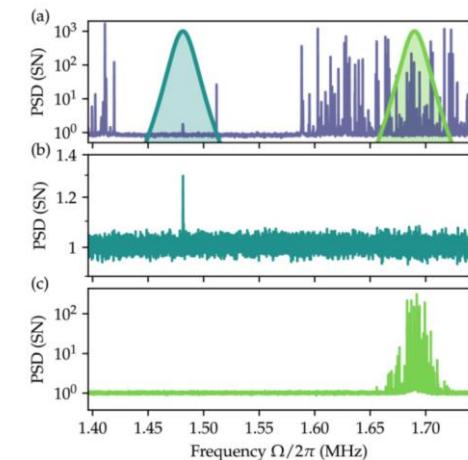
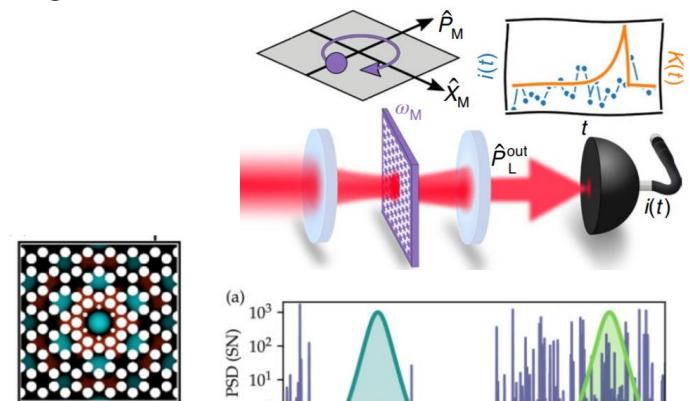
Nature 485, 482–485(2012)

Hot and cold atoms MHz-kHz



Optica 1, pp. 84-88 (2014)

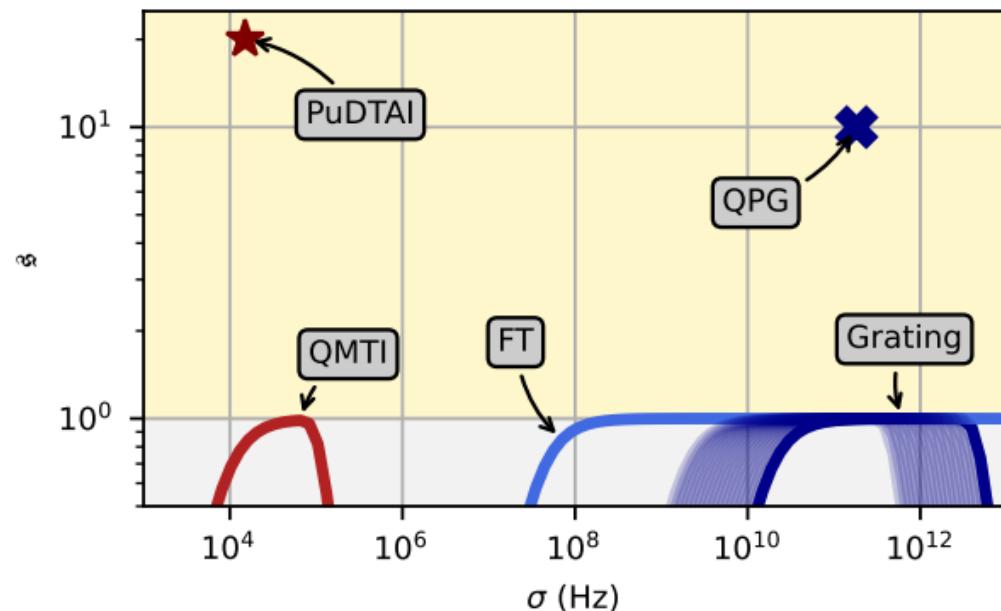
Optomechanical systems
~10kHz



Optica 7, 718-725 (2020)

Comparison

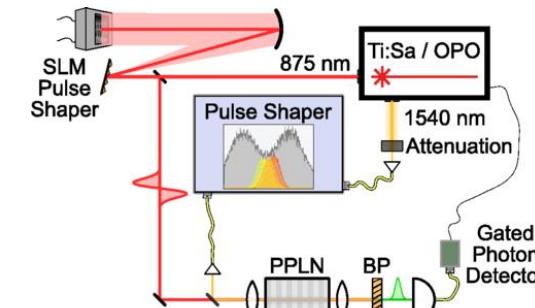
Our approach: PuDTAI



Superresolution parameter:

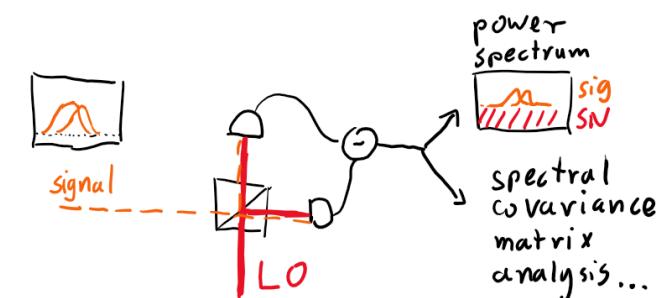
$$s = \lim_{\varepsilon \rightarrow 0} (\mathcal{F}/\mathcal{F}_{DI})$$

Quantum Pulse Gate (QPG) - SPADE



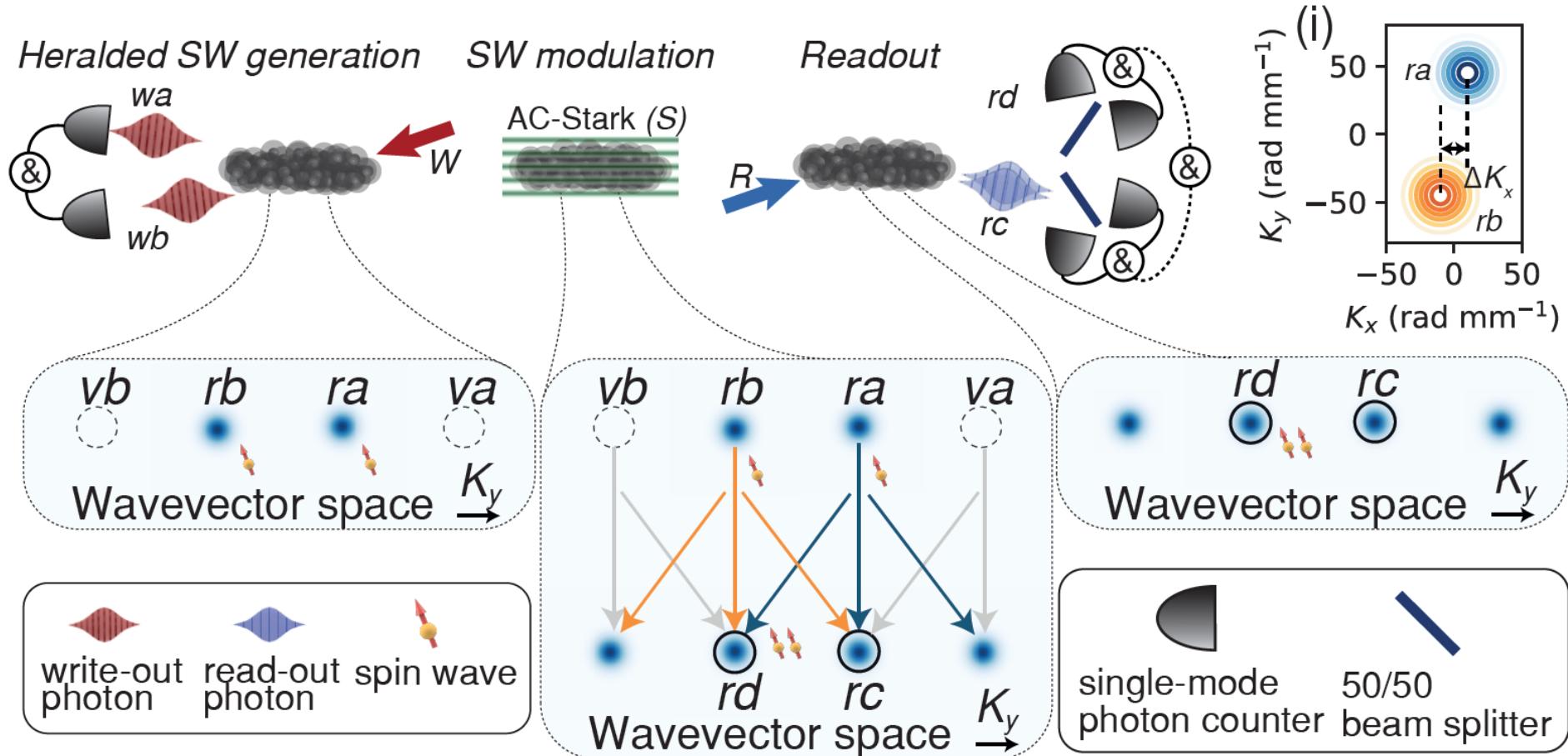
Phys. Rev. Lett. 121, 090501 (2018)

Homodyne/Heterodyne (under development)

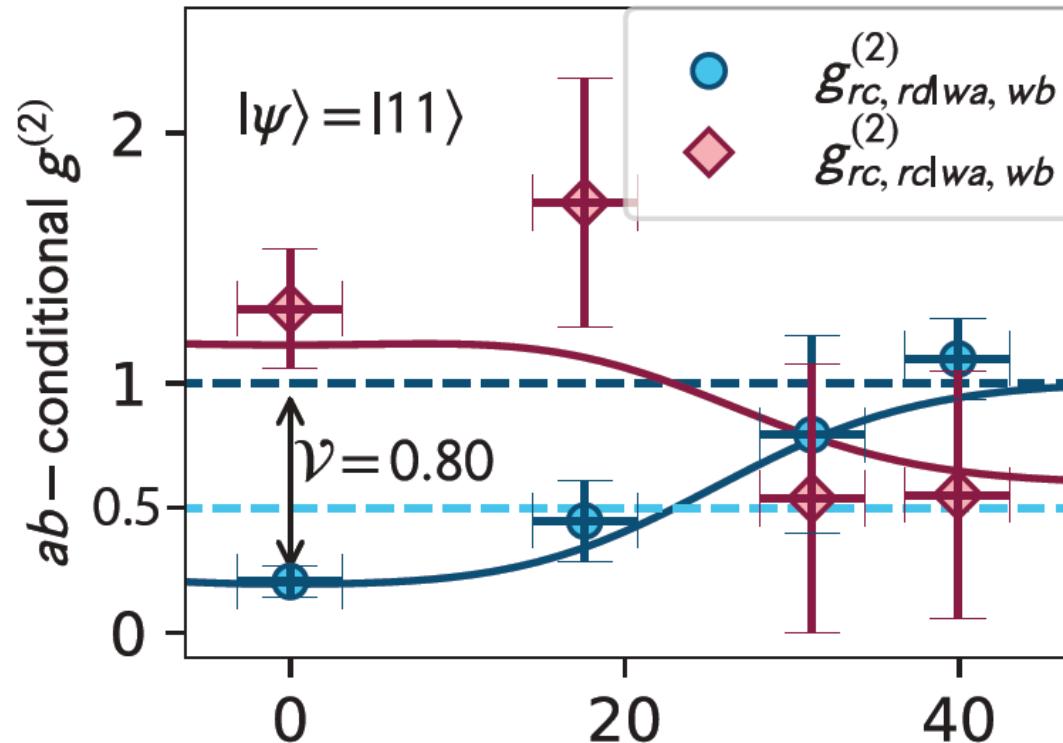
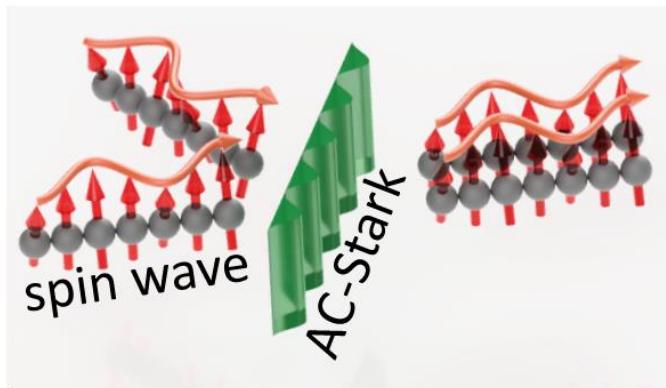
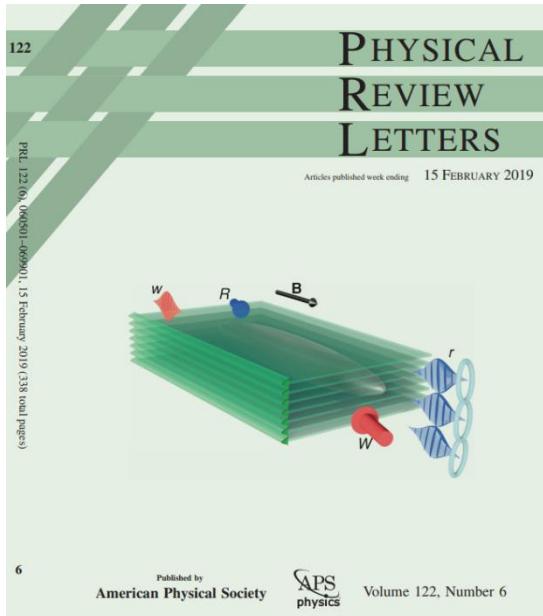


Phys. Rev. A 102, 063526 (2020)

The three-way splitter



Hong-Ou-Mandel interference

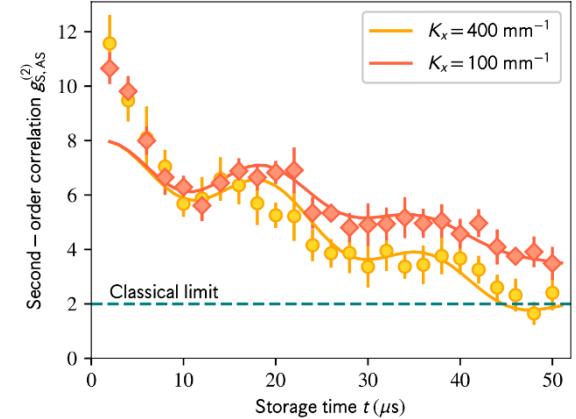
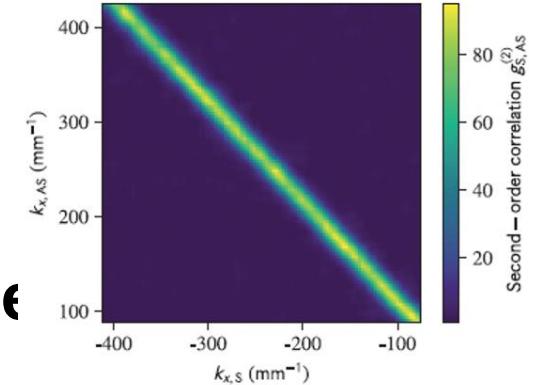
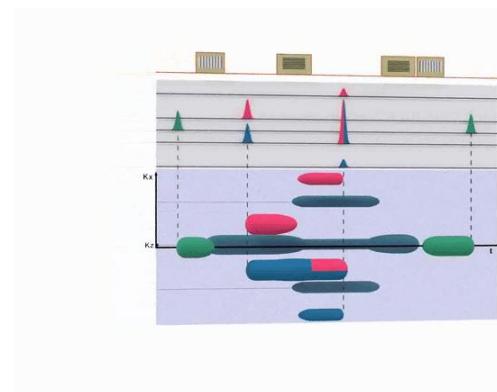
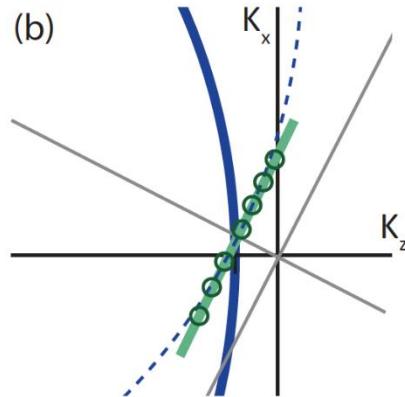
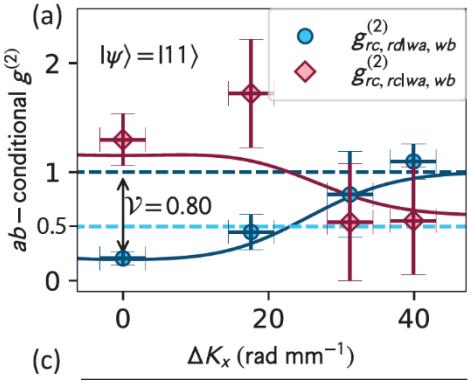


80% visibility at the moment limited by photon purity: with weak coherent states we observed interferometric visibility of >95%

Phys. Rev. Lett. **122**, 063604 (2019)

Atom-embedded photonic (co)processor

- Wavevector-multiplexed quantum memory
- Spin-wave-based interferometric processor for storage
- Multiplexed quantum repeaters

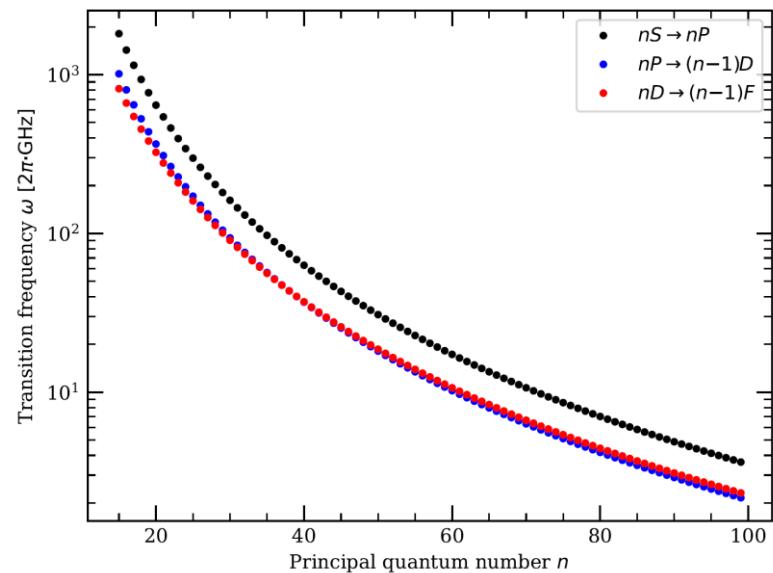


Rydberg atoms

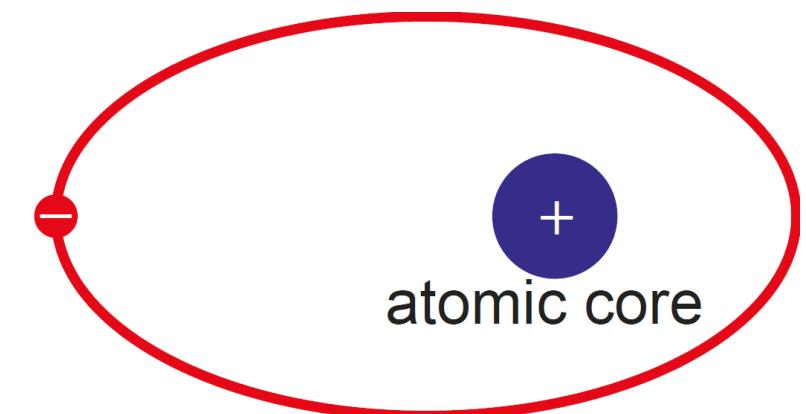


Rydberg atoms

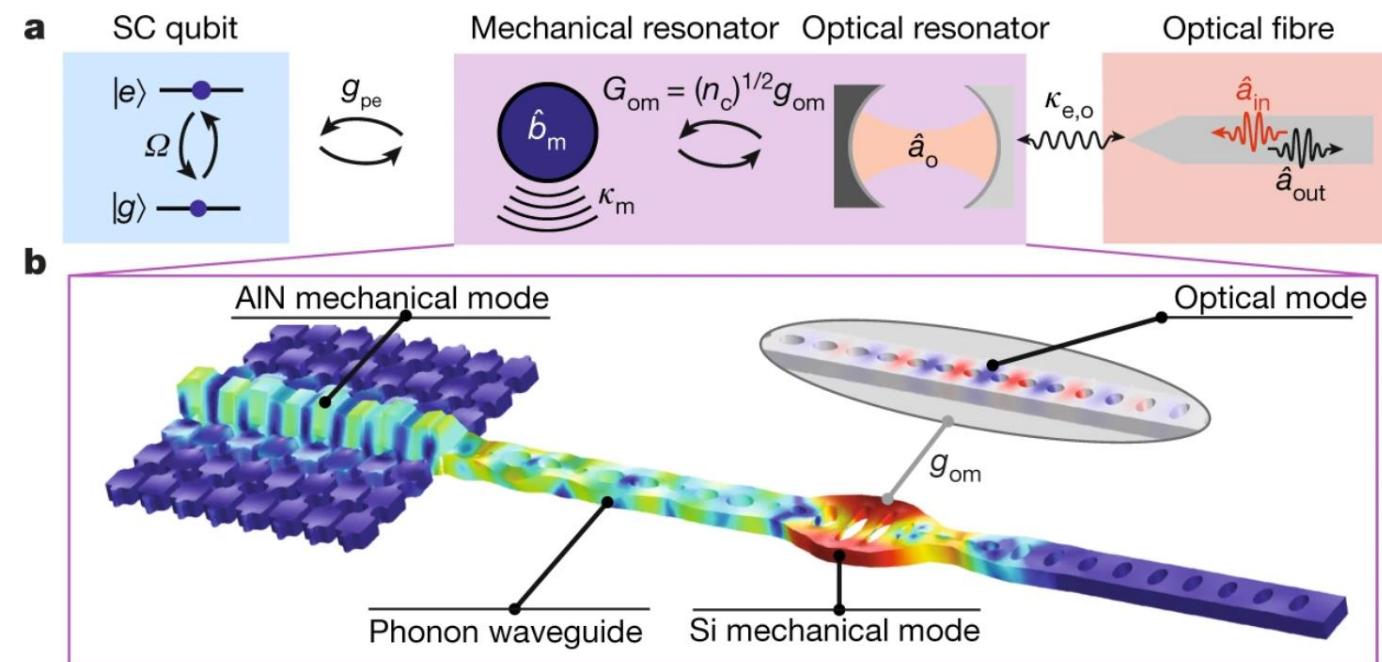
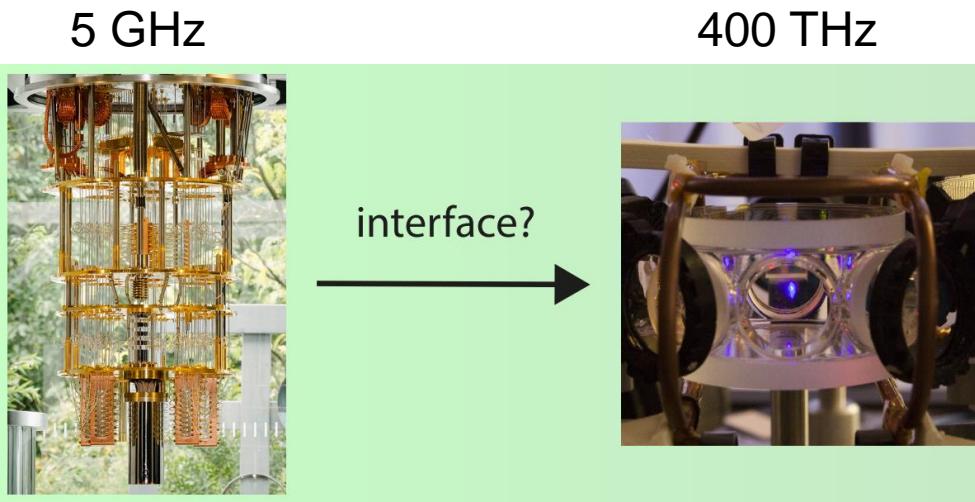
- large principal quantum numbers $n \approx 15-100+$
- long state lifetimes $\tau \approx 15-200 \mu\text{s}$
- long interaction radii $\sim \mu\text{m}$
- microwave (and mm-wave) energy levels transitions $\Delta f \approx 1000-2 \text{ GHz}$



valence
electron

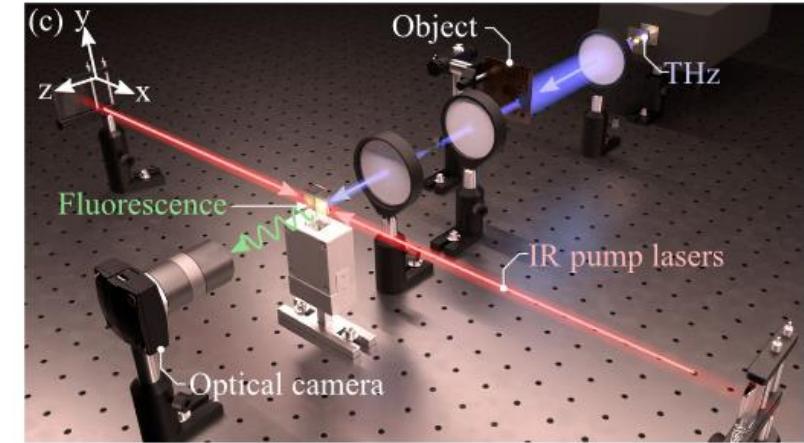
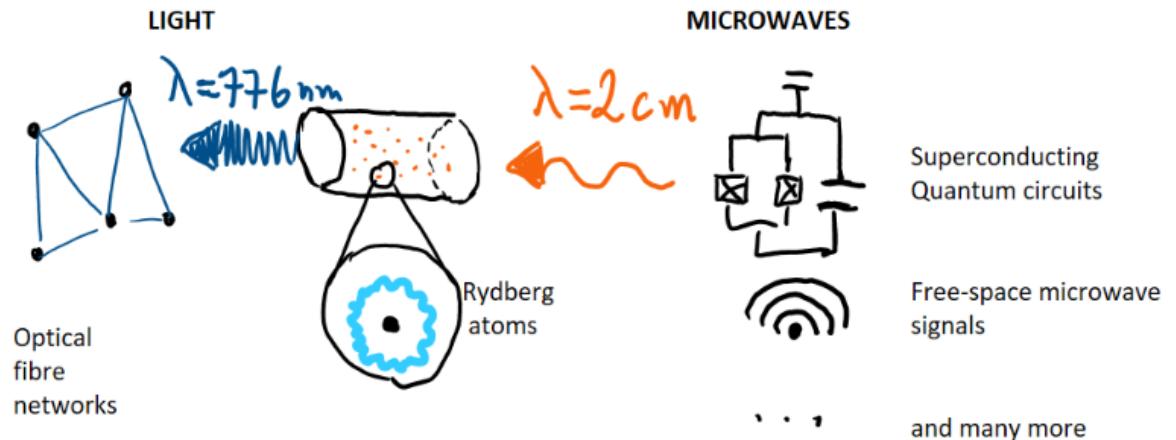


Microwave-optical interface

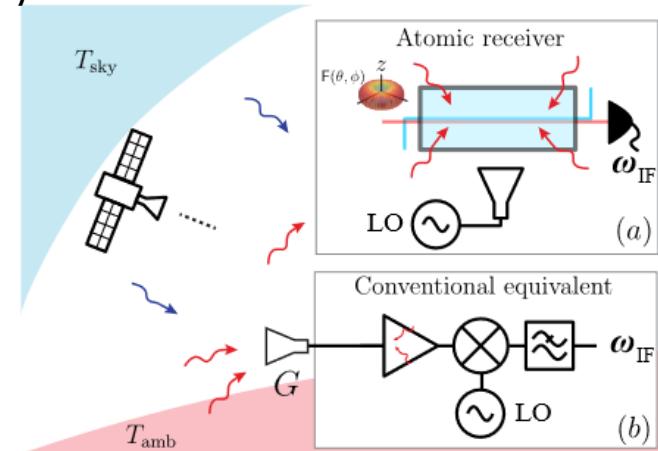


M. Mirhosseini et al. *Nature* **588**, 599–603 (2020)

Applications of quantum transduction



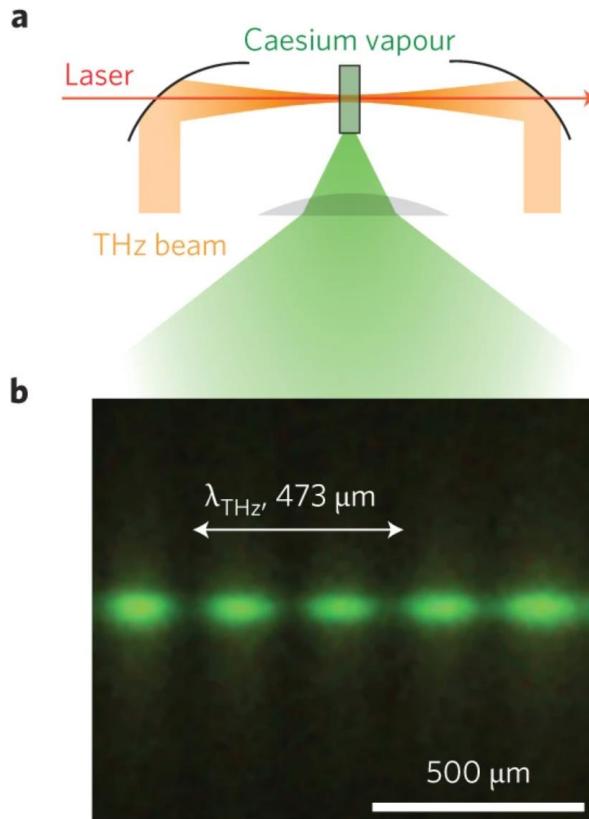
L. A. Downes et al., Phys Rev X 10, 011027 (2020)
(Durham)



G. Santamaria Botello et al., arXiv:2209.00908
(CU Boulder)

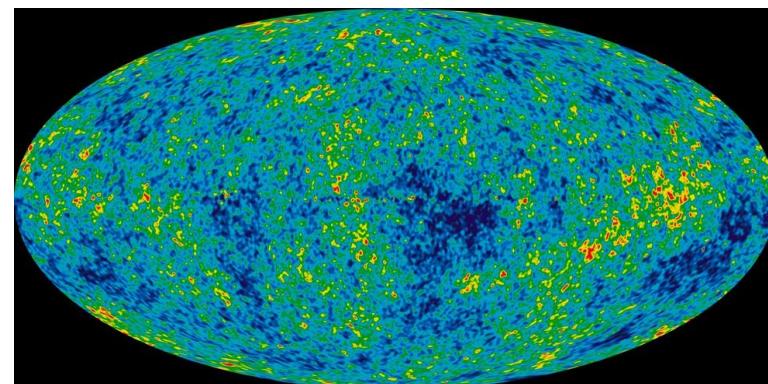
Microwave detection

GHz & THz imaging



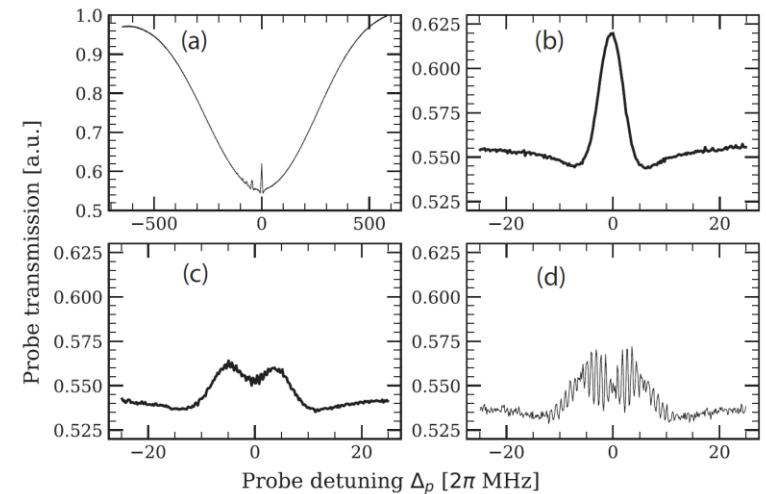
C.G. Wade et al. *Nat. Phot.* **11**, 40–43 (2017)

Microwave astronomy



source:
https://upload.wikimedia.org/wikipedia/commons/3/3c/llc_9yr_moll4096.png

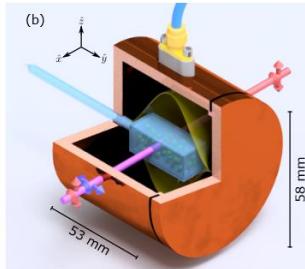
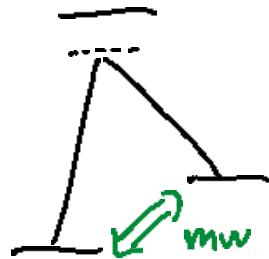
Next-gen microwave sensing and communication



S. Borówka et al. *Appl. Opt.* **61** (29), 8806-8812 (2022)

Other approaches - examples

A system

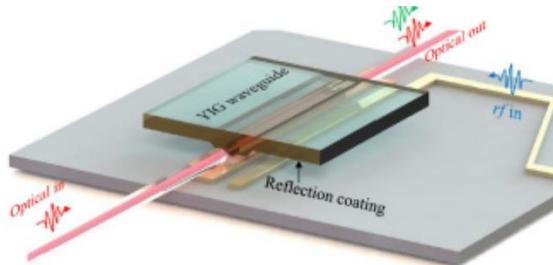


(very low dipole moment)

Ground state atoms, RE ions, ...

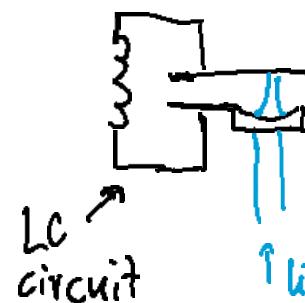
2305.19221 (Alberta, Canada)

Opto-magnonics



Optica 7, 10, 1291 (2020)
(Yale)

opto-electro-mechanics



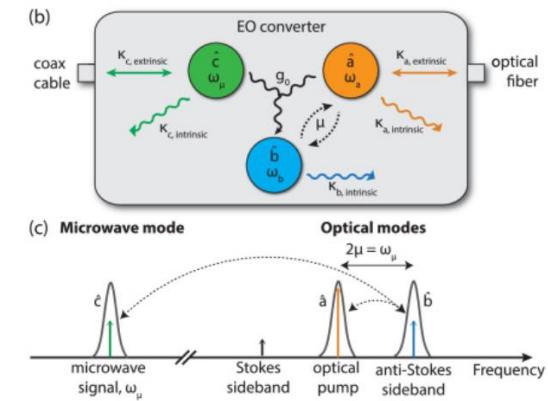
$$\Omega_m = \Omega_{LC}$$

(very complex)

Si or SiN resonators, ...

Nature Phys. 16, 69–74 (2020)
(Delft)

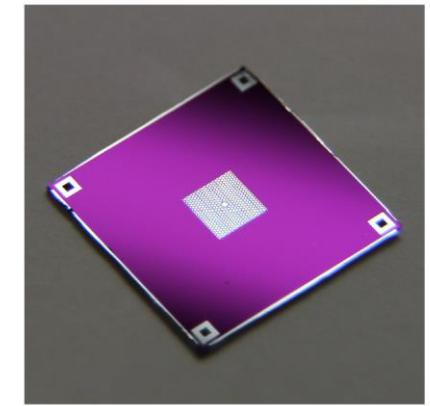
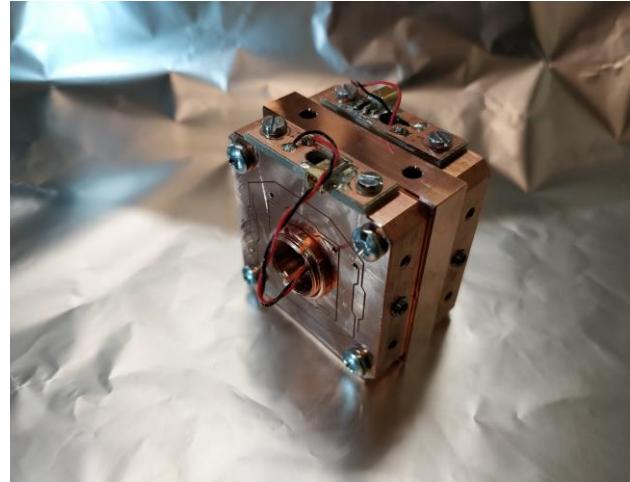
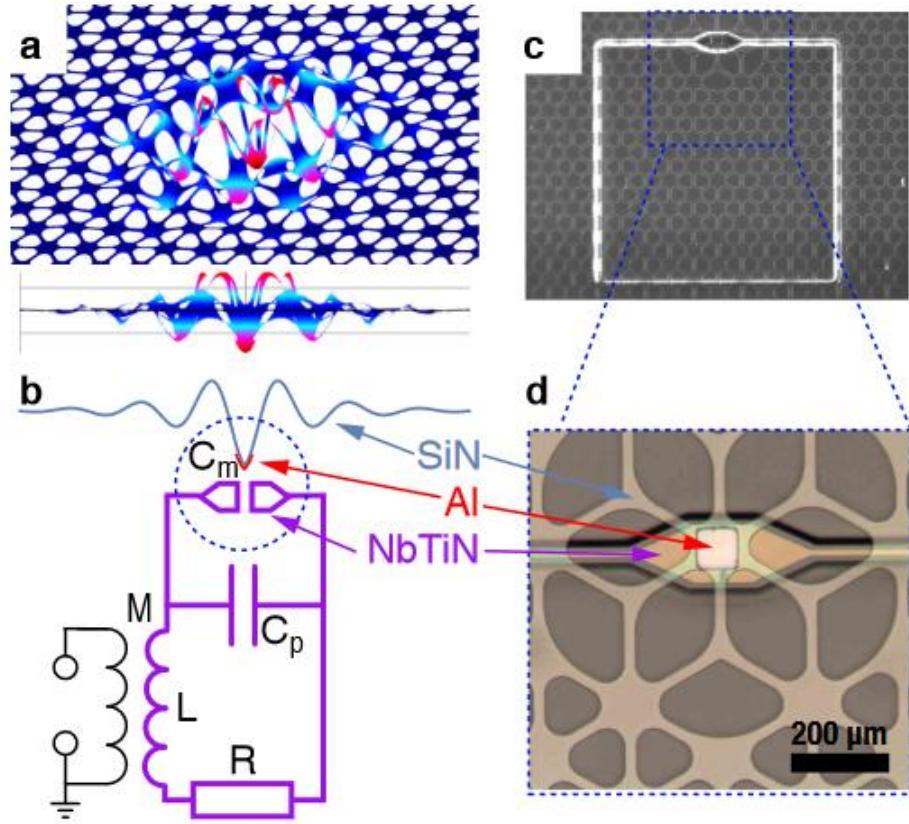
Electro-optics



LN resonator, ...

Optica 7, 12, 1737-1745 (2020)
(Stanford)

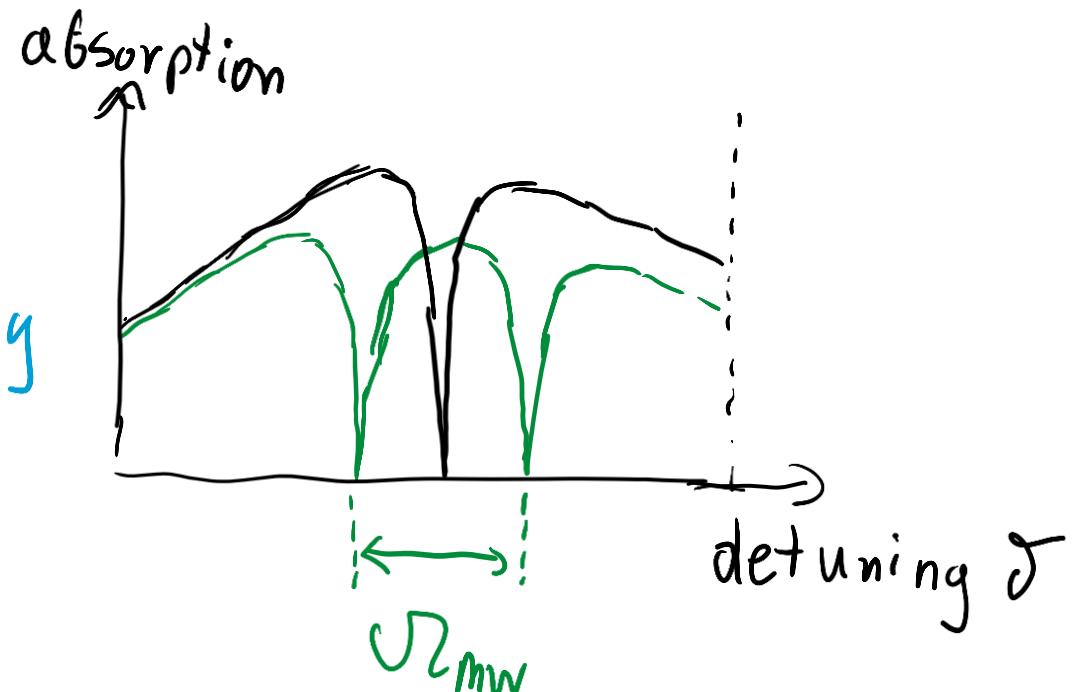
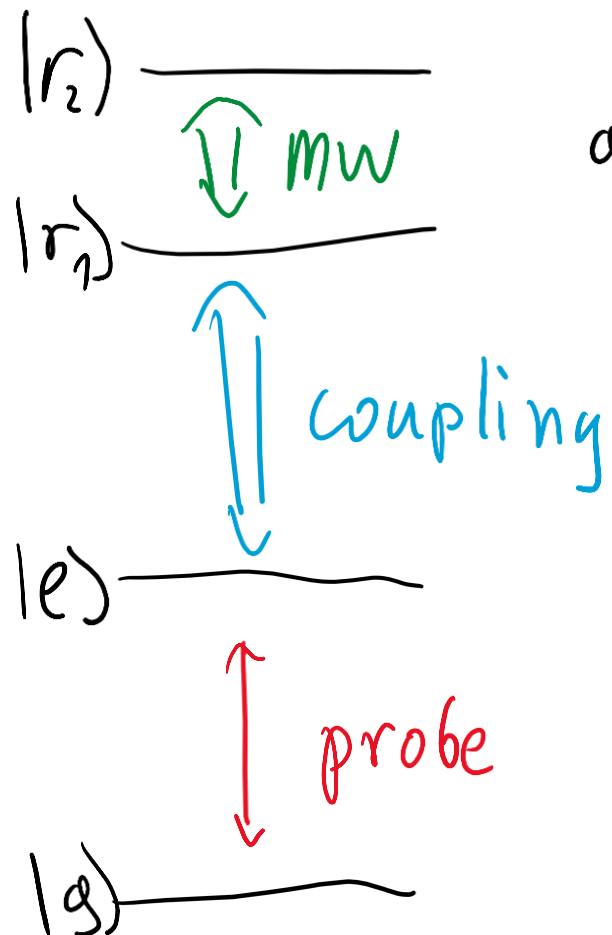
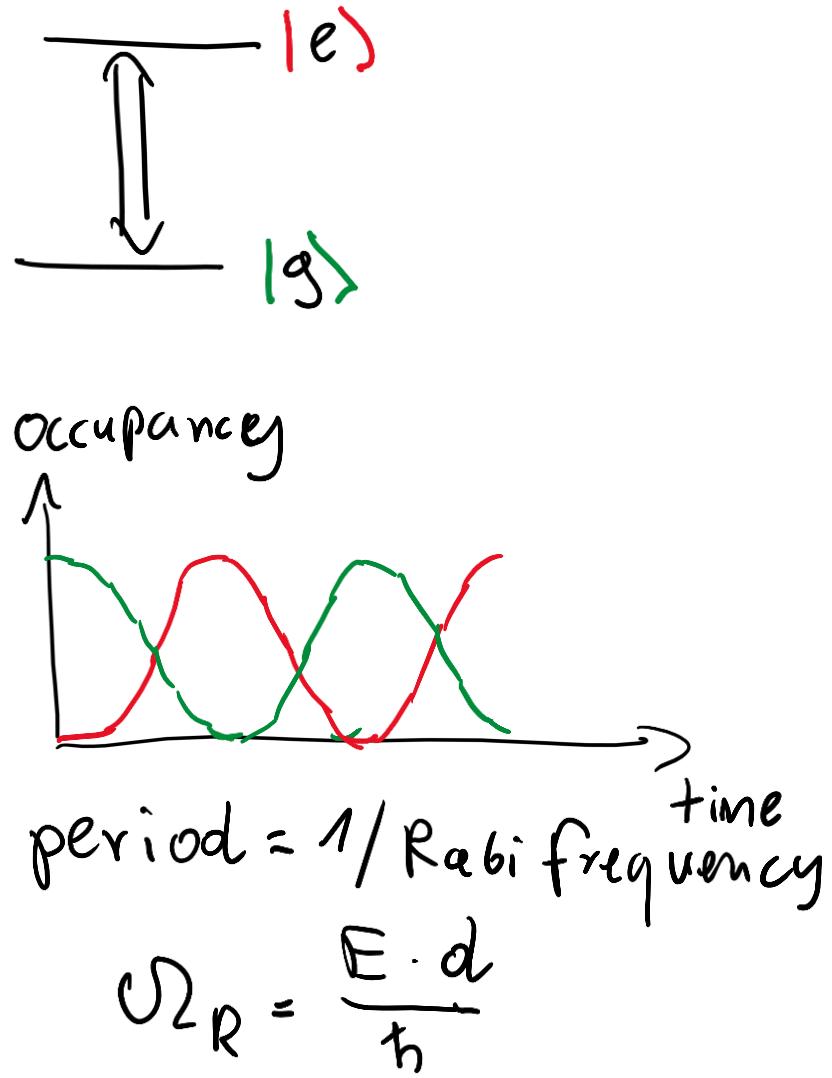
Other approaches



Nat. Commun. 13, 1507 (2022) (NBI Copenhagen)
A. Schliesser

I. Galinskiy, Y. Tsaturyan, MP, E. S. Polzik, Optica 7, 718 (2020)
R.A. Thomas, MP, et al., Nature Physics 17, 228–233 (2021)
I. Galinskiy, G. Enzian, M. Parniak, E. S. Polzik, Phys Rev. Lett. 133, 173605 (2024)

Rabi frequency and EIT sensing

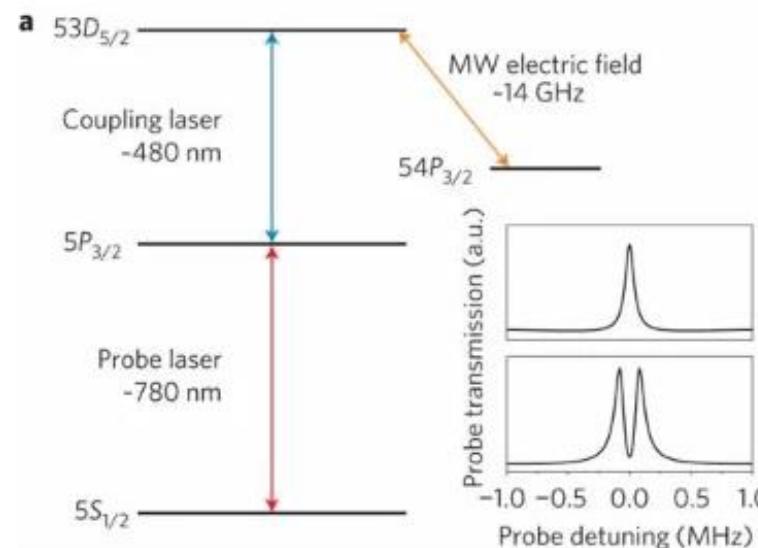


Rydberg electrometry

Frequency splitting

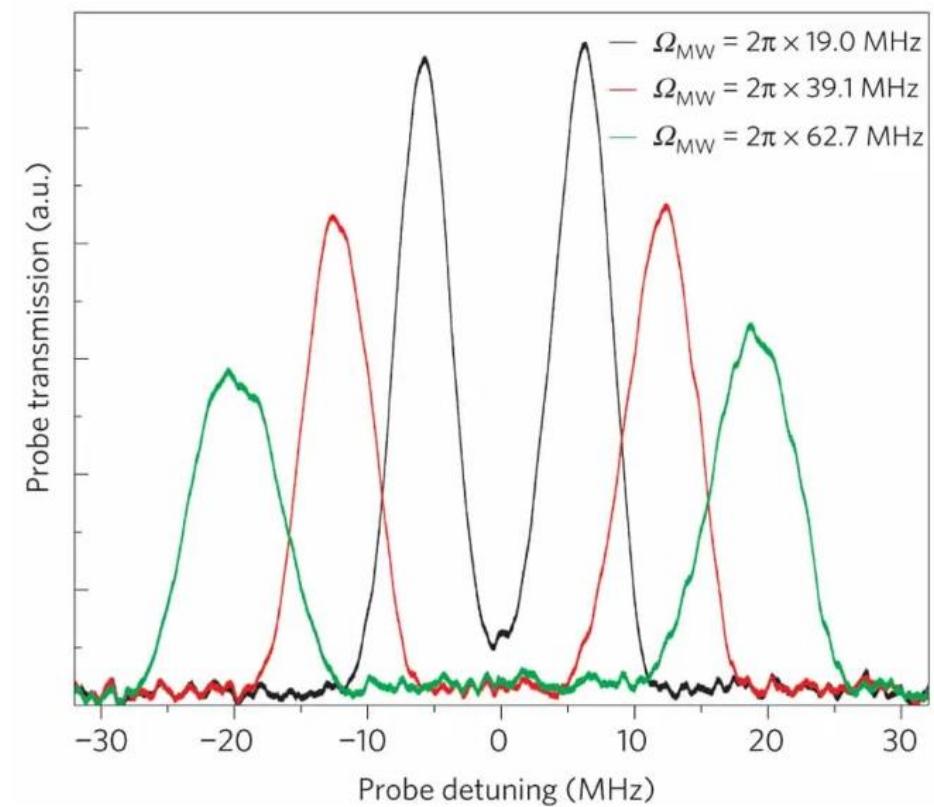
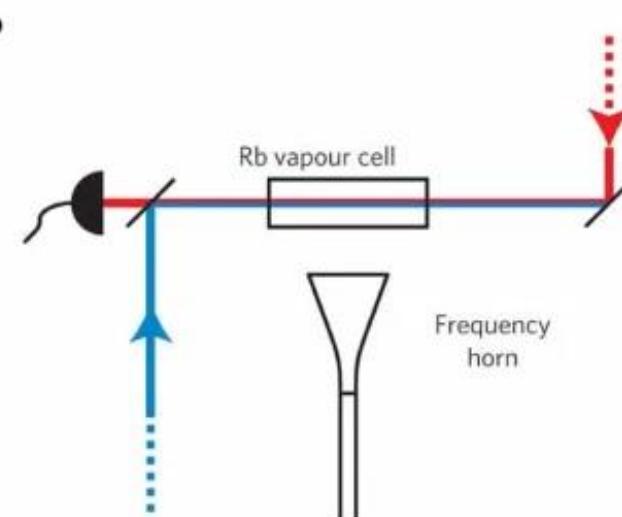
- Rabi frequency -

measured with respect to a frequency reference

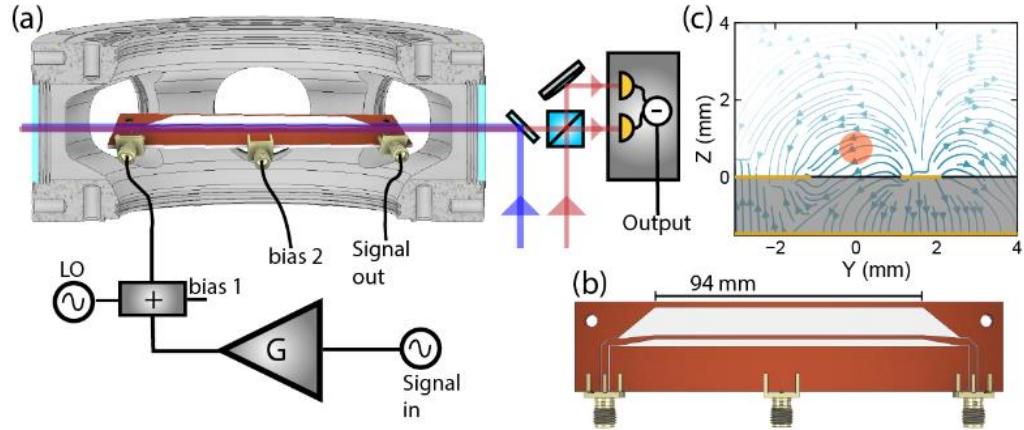


$$\Omega = \frac{d_x E_0}{\hbar}$$

Electric field amplitude E
Dipole moment d



Practical electrometers

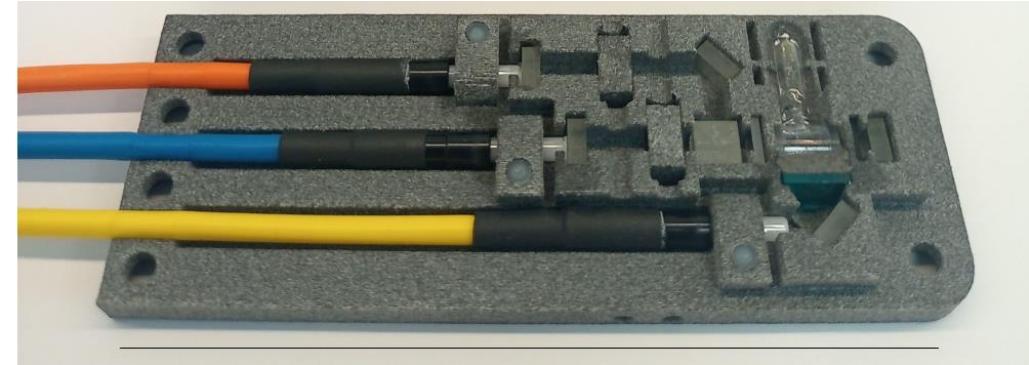


D. Meyer et al., PR Applied **15**, 014053 (2020) (ARL Maryland)



Soon: smaller cel

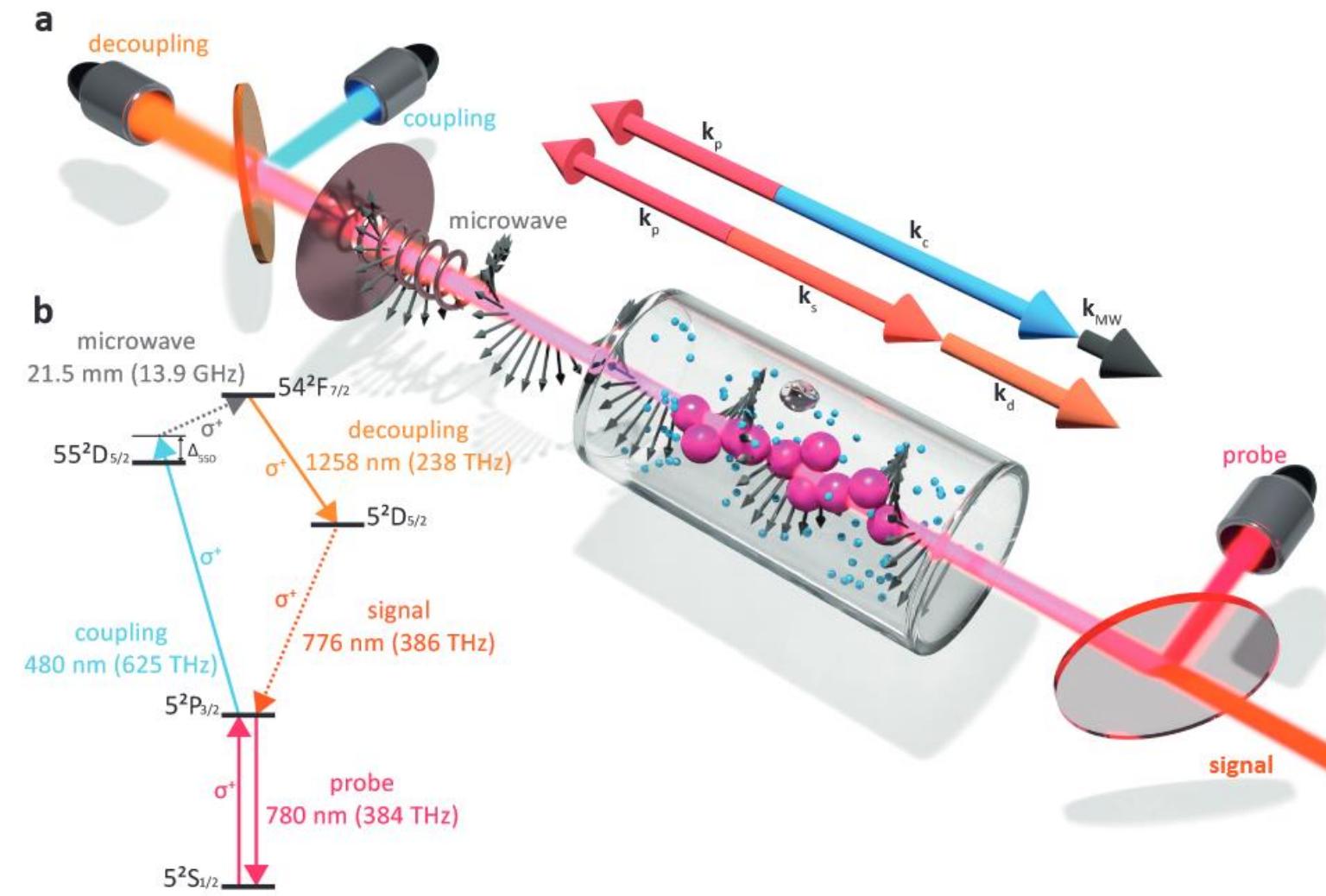
Potential: all-glass/fiber/plastic microwave receiver, insensitive to EMI



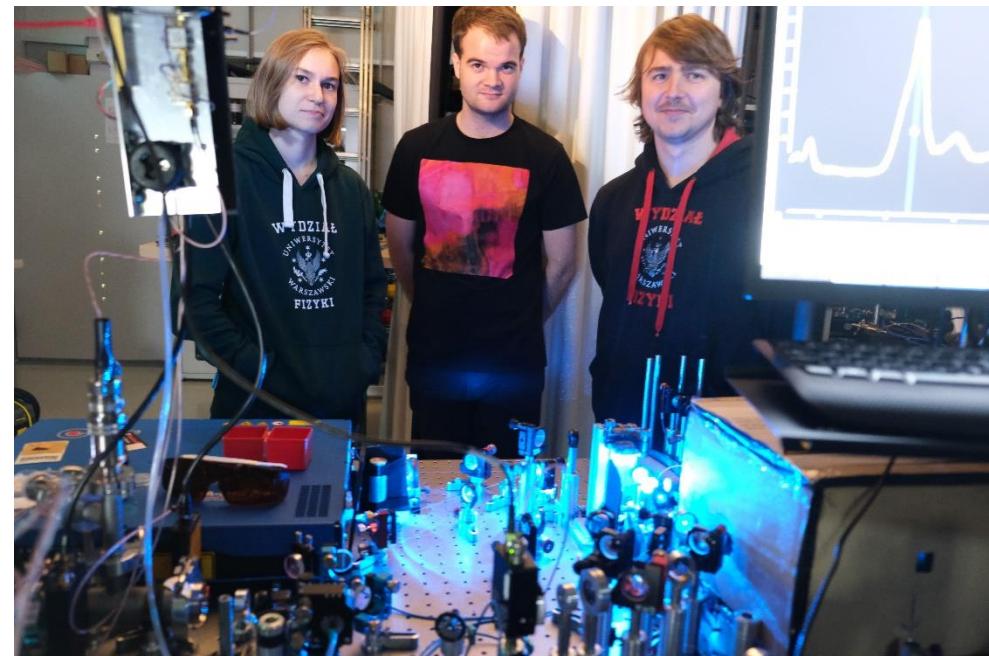
Future: all-fiber laser system?

Example atomic magnetometer by our collaborators from University of Copenhagen
H. Staerkind et al., arXiv:2208.00077 (2022)

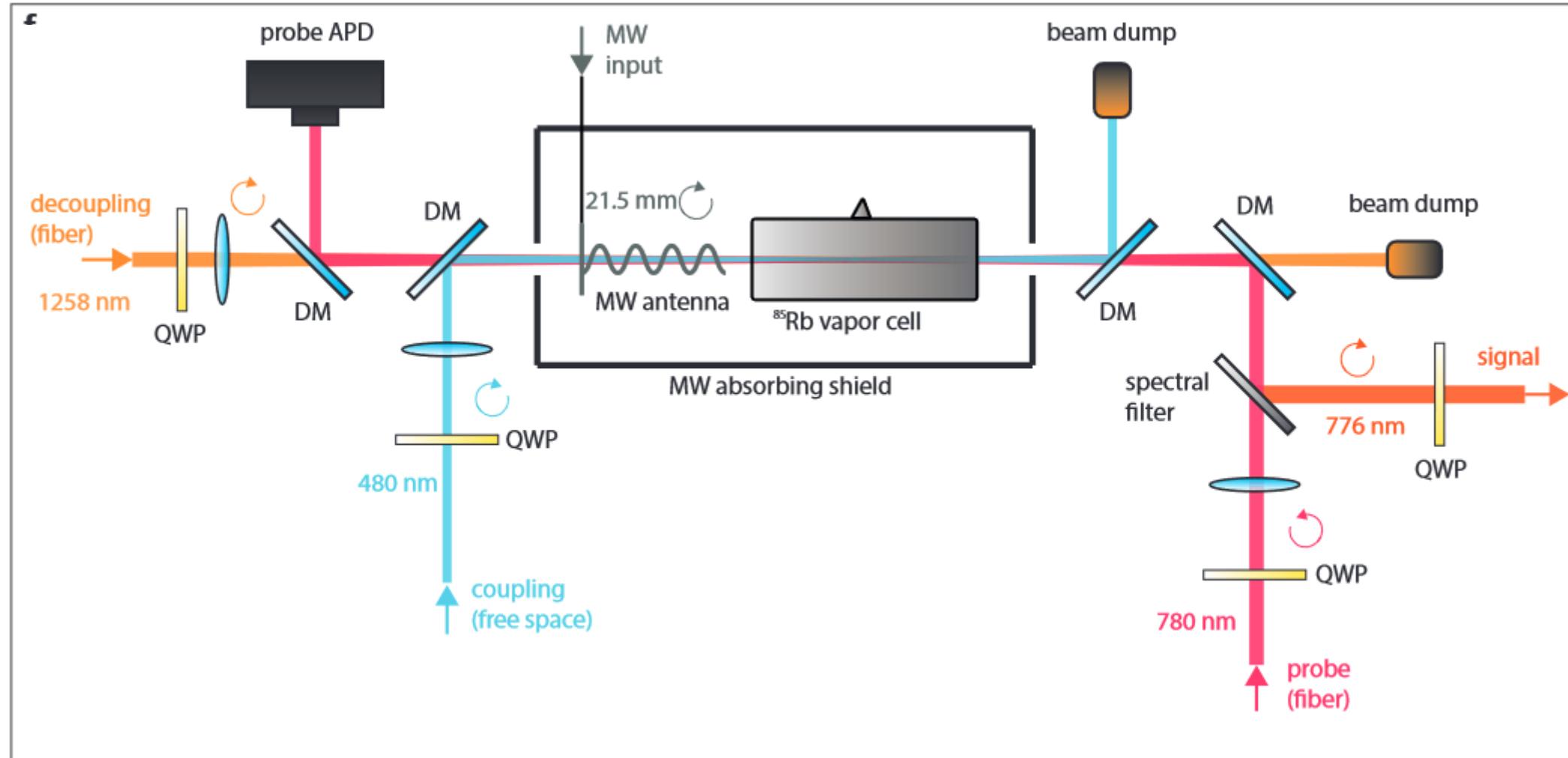
Microwave-to-optical conversion



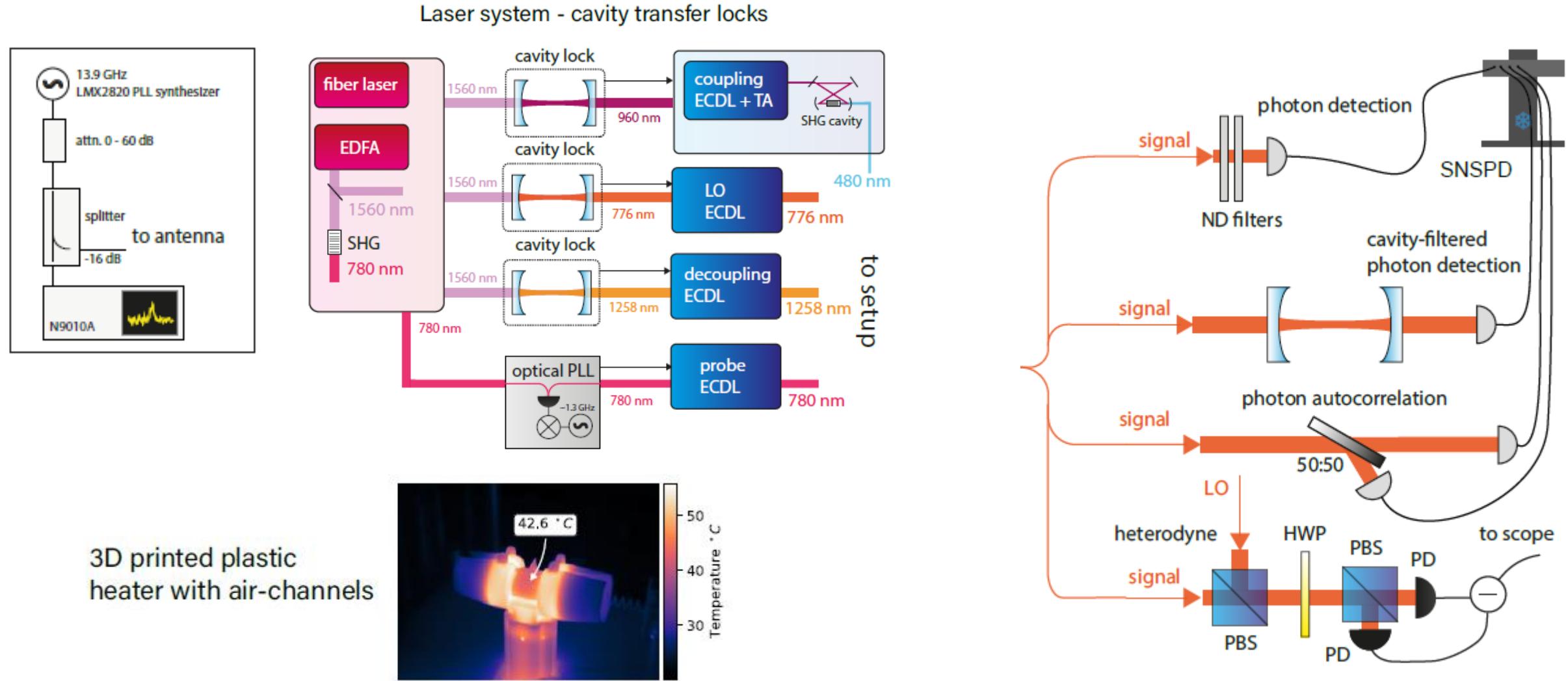
S. Borówka,
U. Pylypenko, M. Mazelanik, **MP**,
Nature Photonics 18, 32-38 (2024)



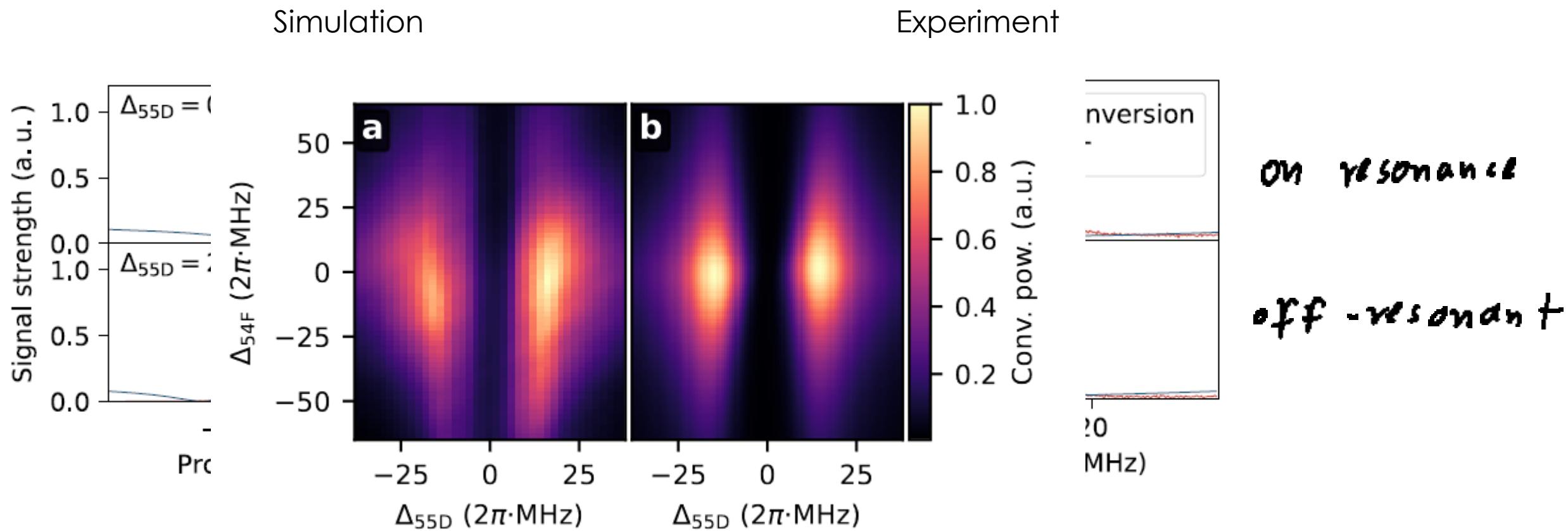
Experimental setup



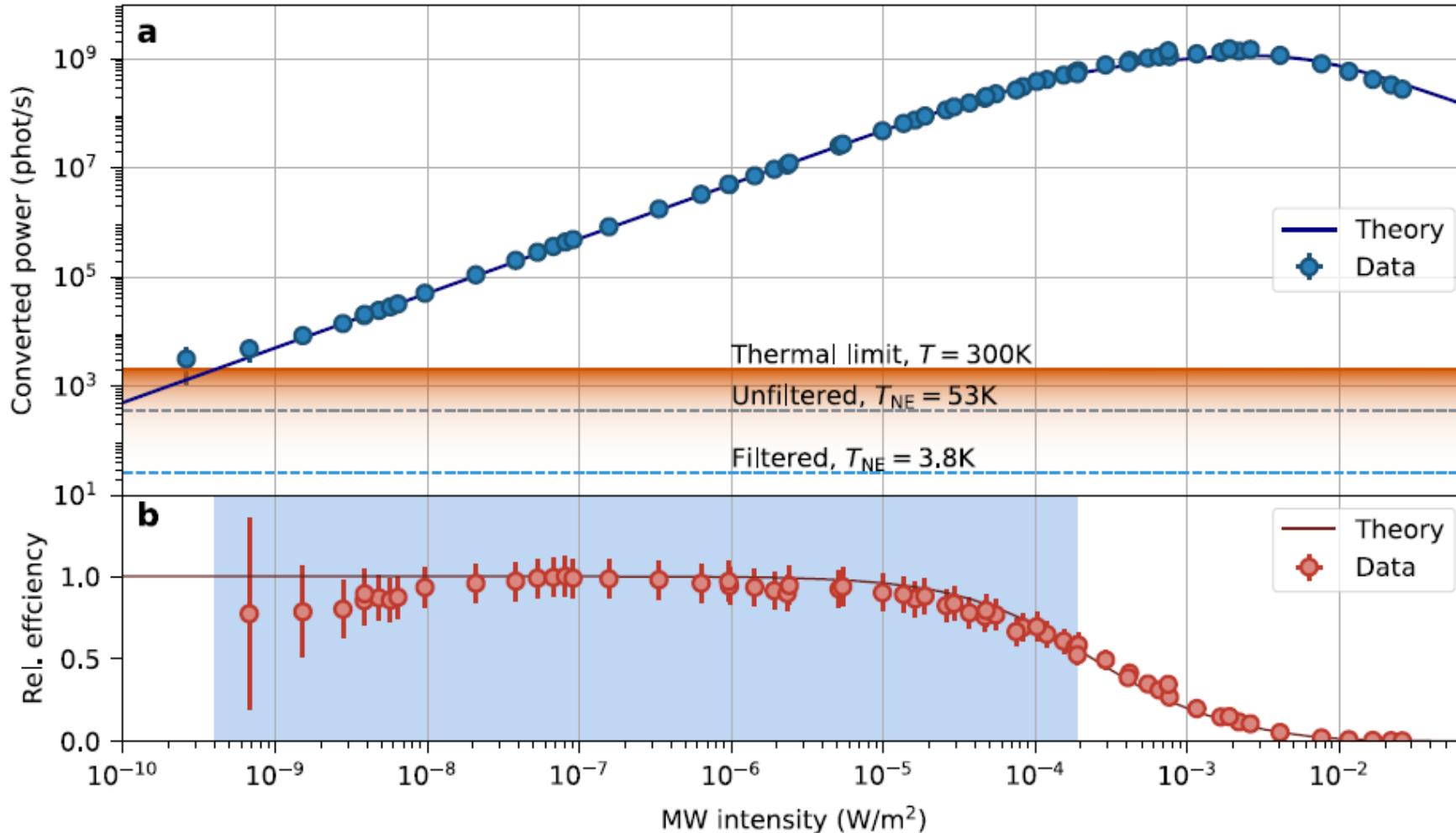
Experimental setup



EIT & conversion



Photon counting and efficiency



Approaching the thermal limit

Theoretical

$$4.11 \text{ nVcm}^{-1}\text{Hz}^{-1/2}$$

Theory of black-body radiation

$$\langle E_{\text{eff}}^2 \rangle = \frac{\omega^2 \langle \mathcal{E} \rangle}{\pi^2 c^3 \varepsilon_0} \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin(\theta) |\eta(\theta)|^2,$$

$$\langle \mathcal{E} \rangle = \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1},$$

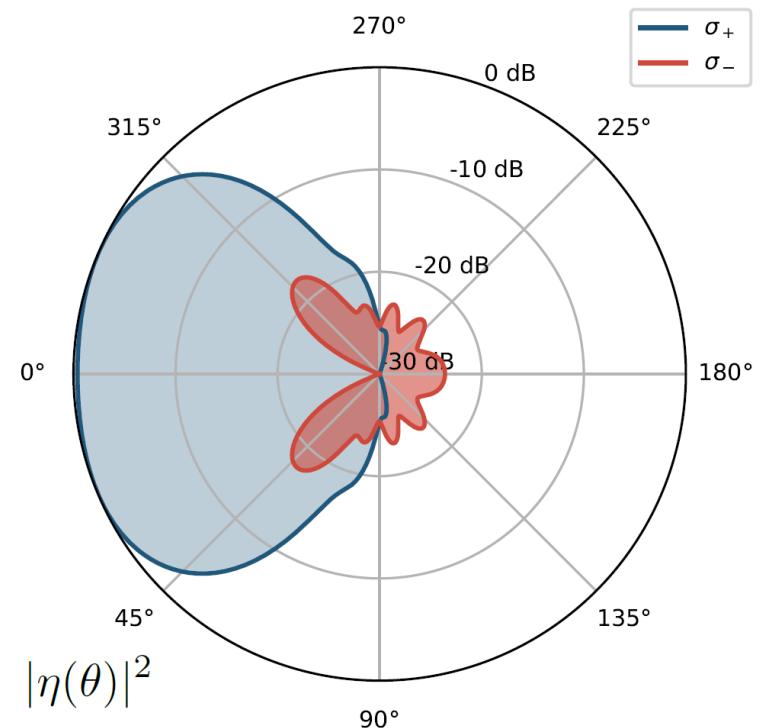
$$|\eta(\theta)|^2 = \left(\cos\left(\frac{\theta}{2}\right)^4 + \sin\left(\frac{\theta}{2}\right)^4 \right) |\eta_{\text{phm}}(\theta)|^2.$$

Polarization

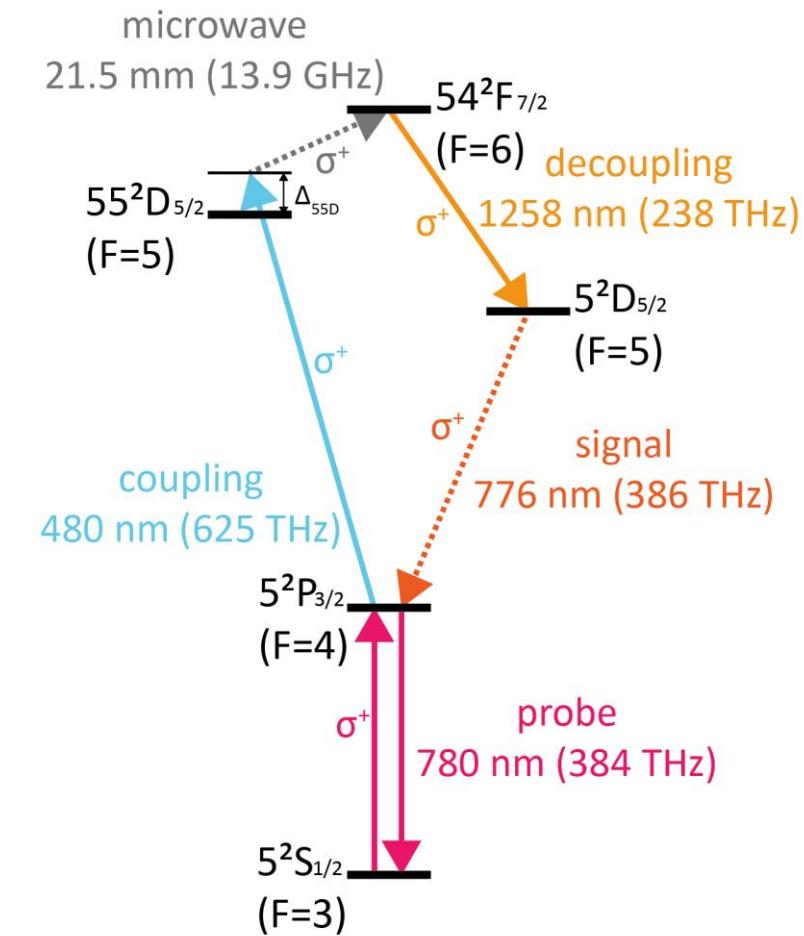
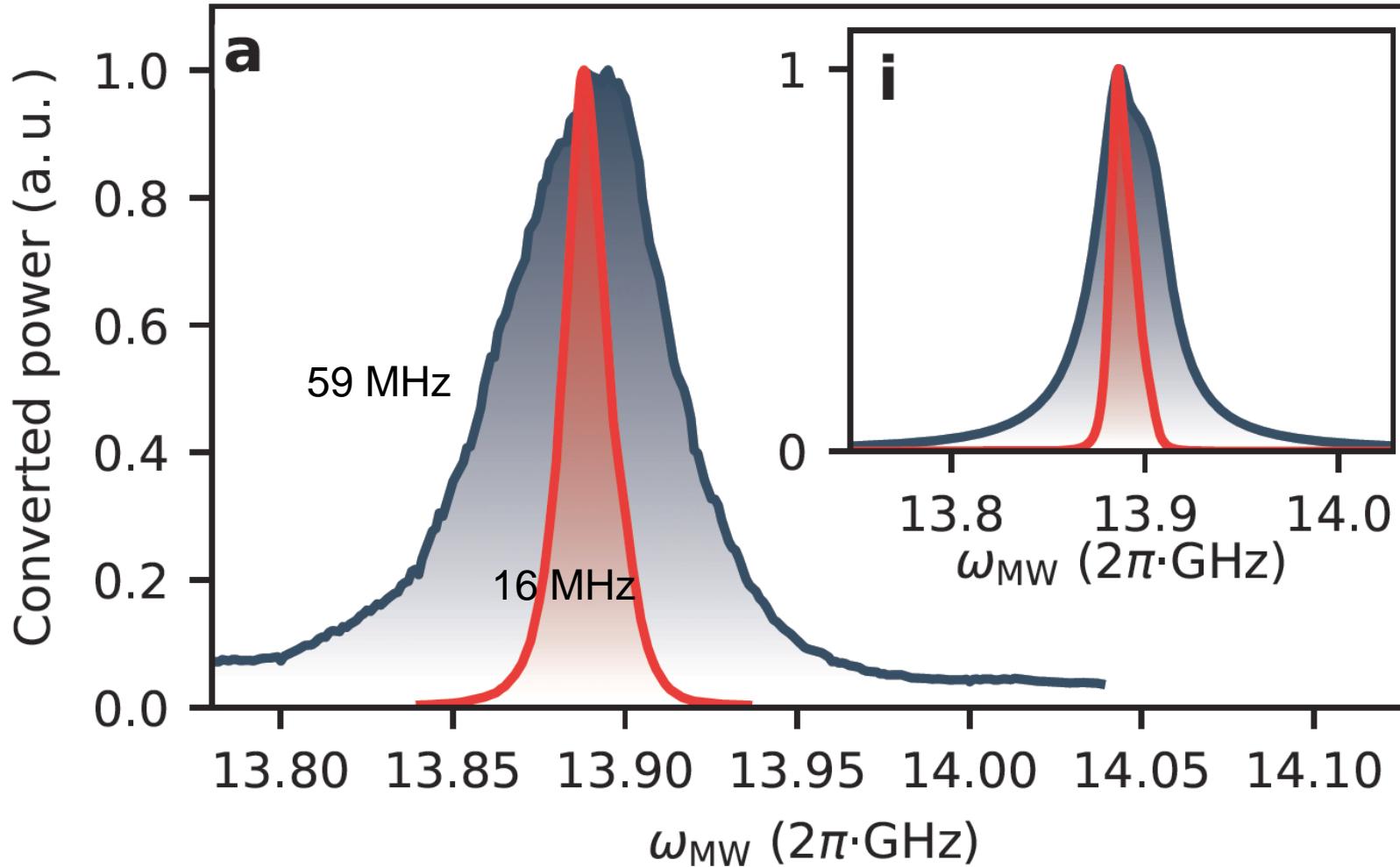
Phase matching

Measured

$$3.98 \text{ nVcm}^{-1}\text{Hz}^{-1/2}$$



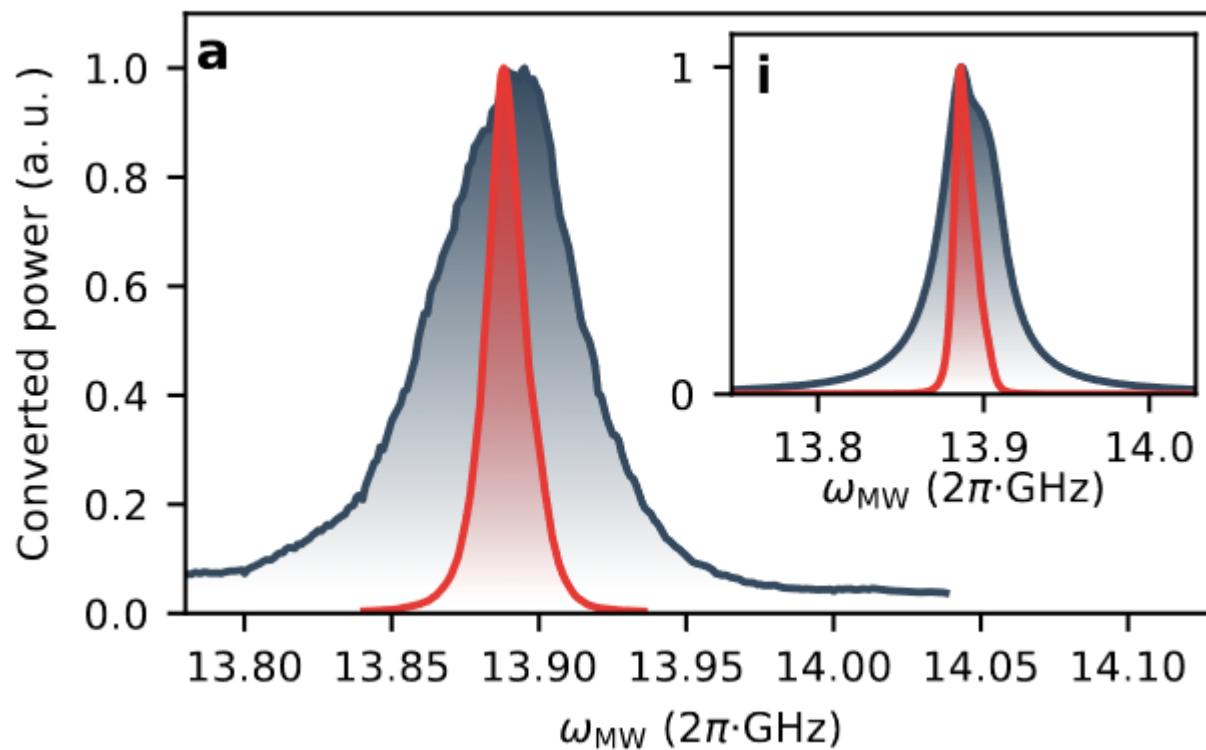
Conversion bandwidth



Bandwidth

Tunable bandwidth (single laser)

Instantaneous bandwidth

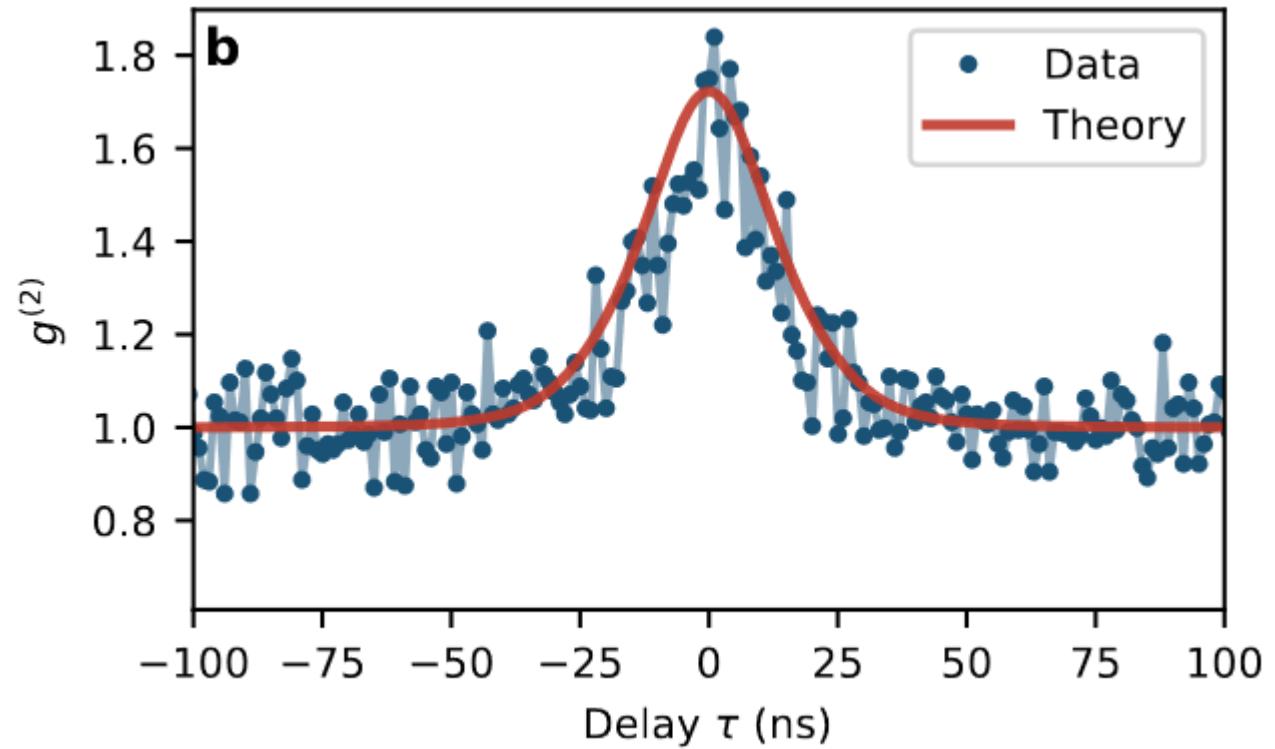
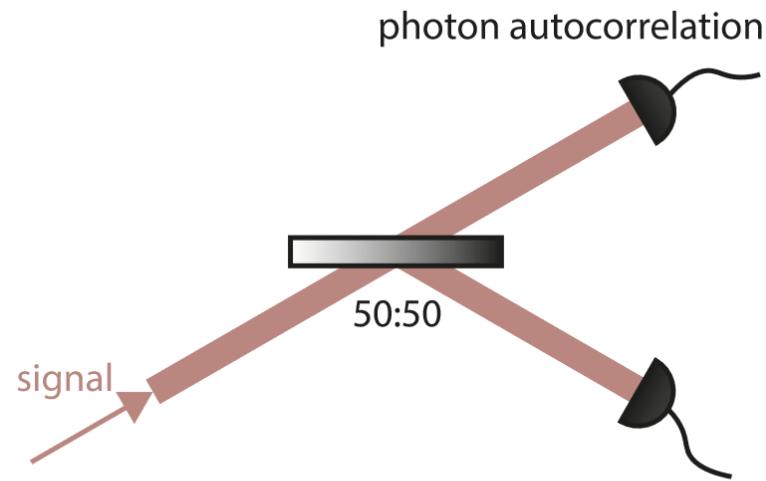


Wiener-Khinchin theorem!

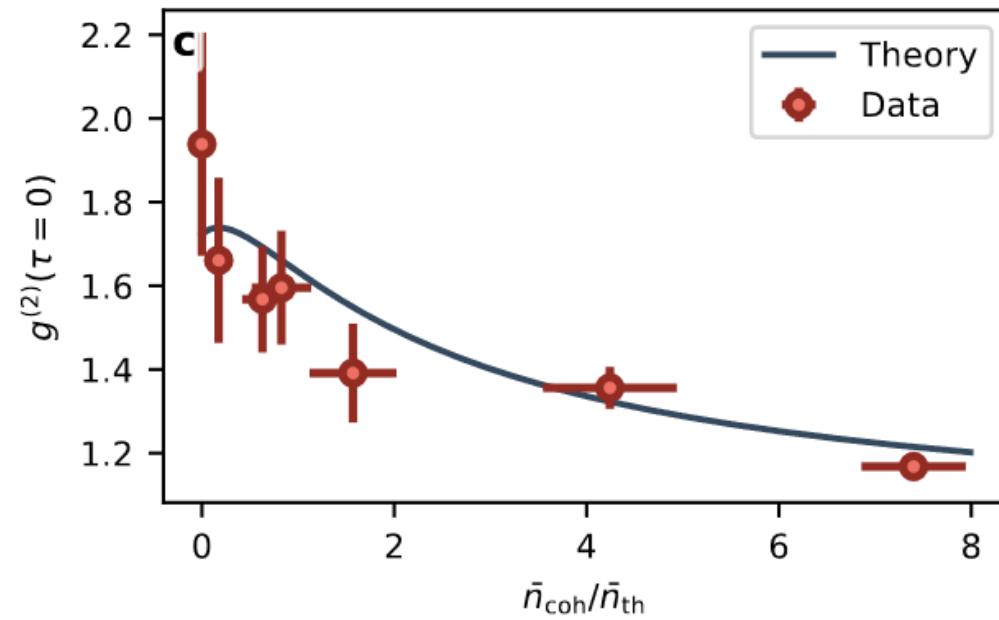
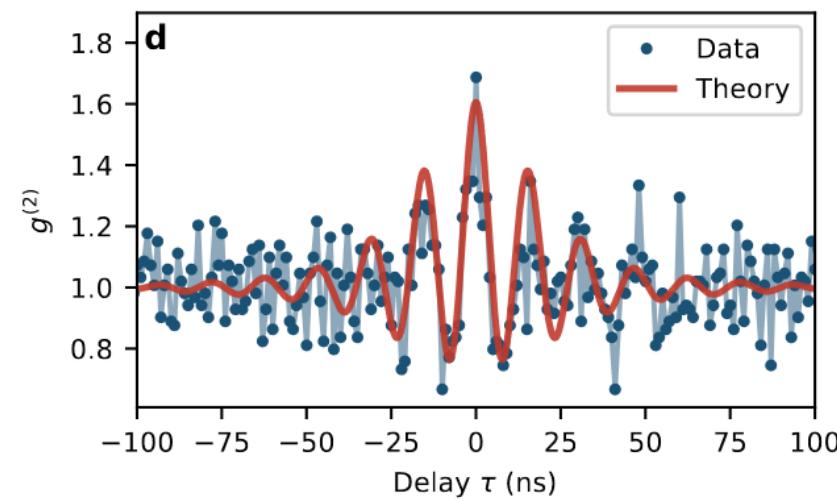
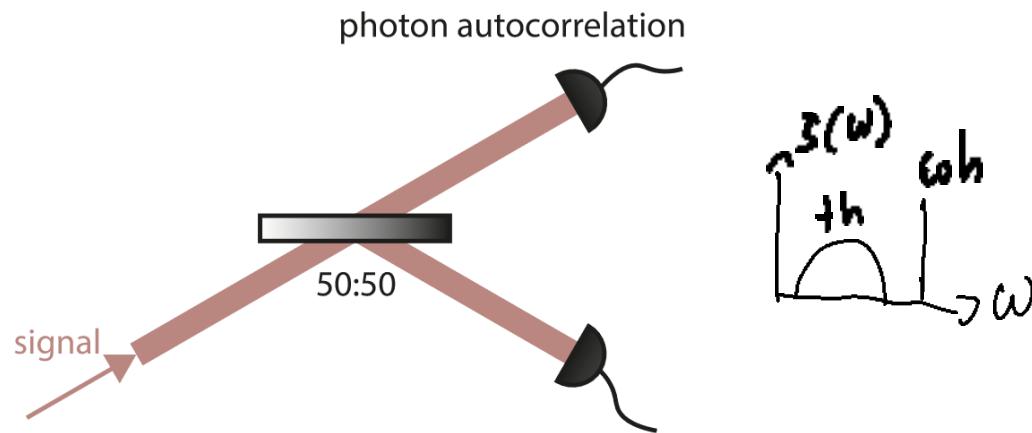
$$g_{\text{th}}^{(1)}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |S(\omega)|^2 e^{-i\omega\tau} d\omega,$$

$$g^{(2)}(\tau) = 1 + \frac{1}{4} \left(\left| g_{\text{th}}^{(1)}(\tau) + e^{-i\omega\tau} \right|^2 - 1 \right),$$

Second-order correlation



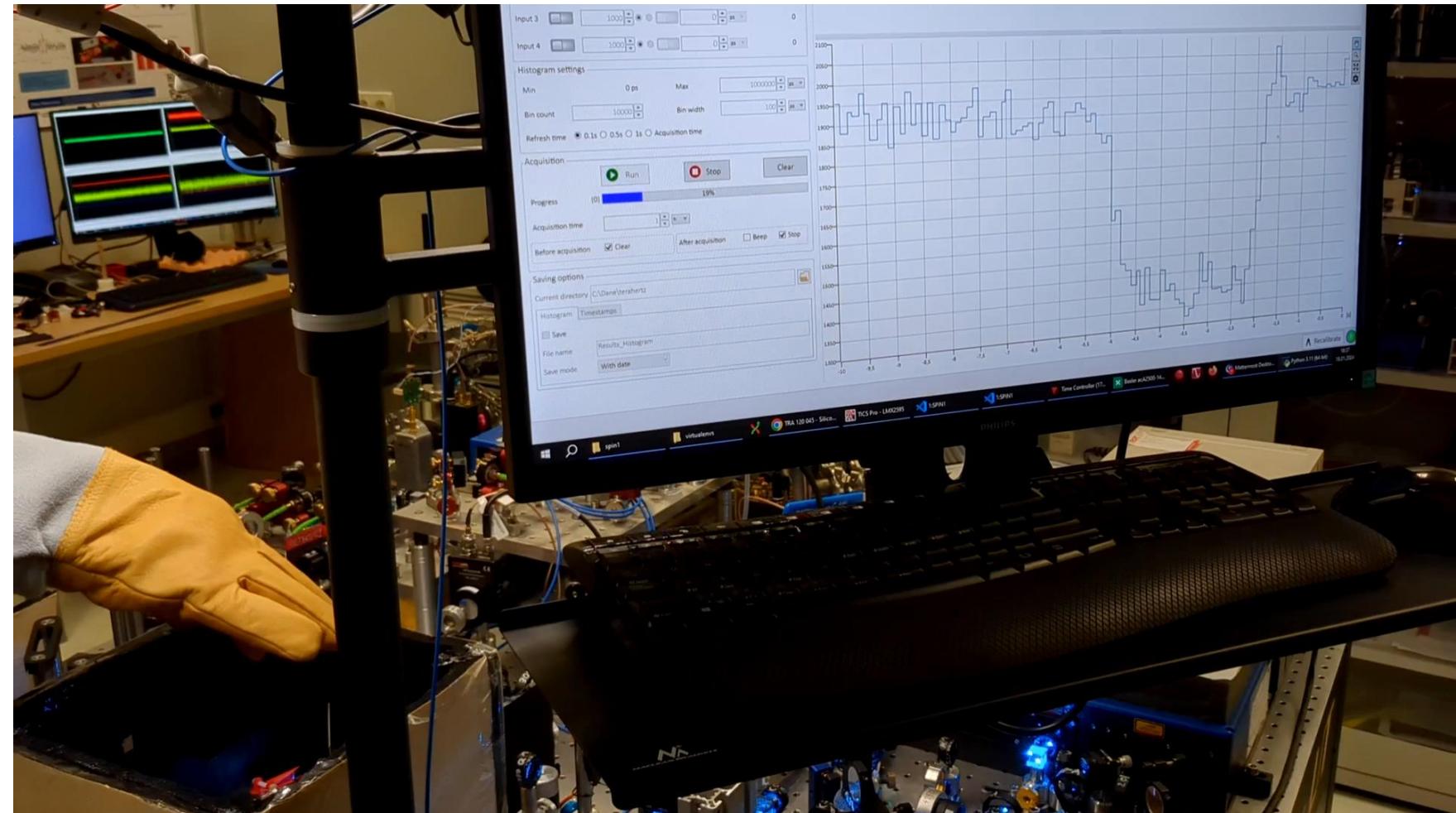
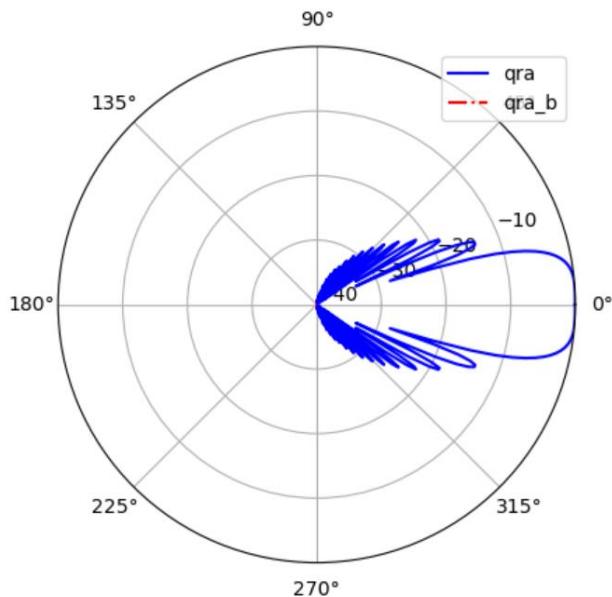
Second-order correlation



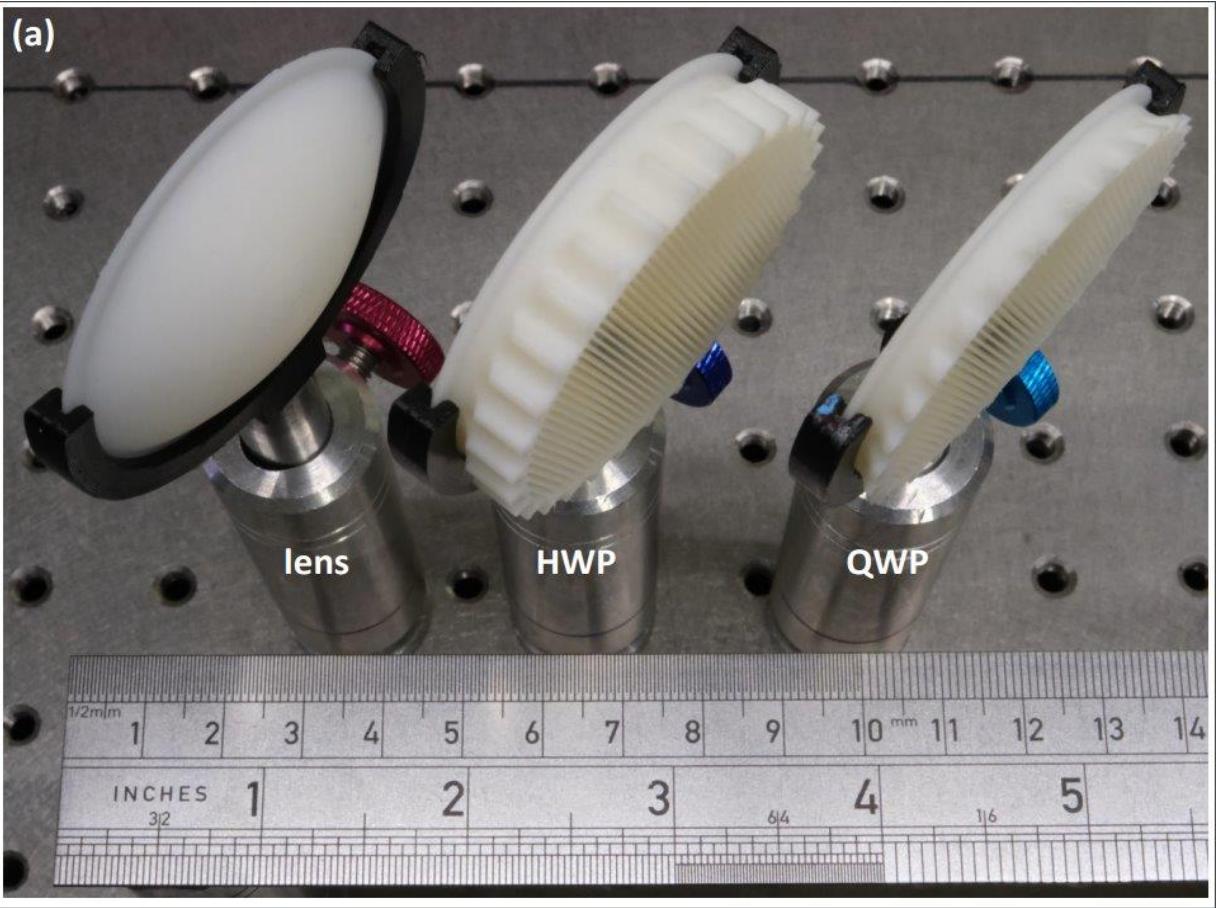
$$g^{(2)}(\tau) = 1 + \frac{\left| \bar{n}_{th}g_{th}^{(1)}(\tau) + \bar{n}_{coh}e^{-i\omega\tau} \right|^2 - \bar{n}_{coh}^2}{(\bar{n}_{th} + \bar{n}_{coh} + \bar{n}_{noise})^2}$$

Conversion @130 GHz

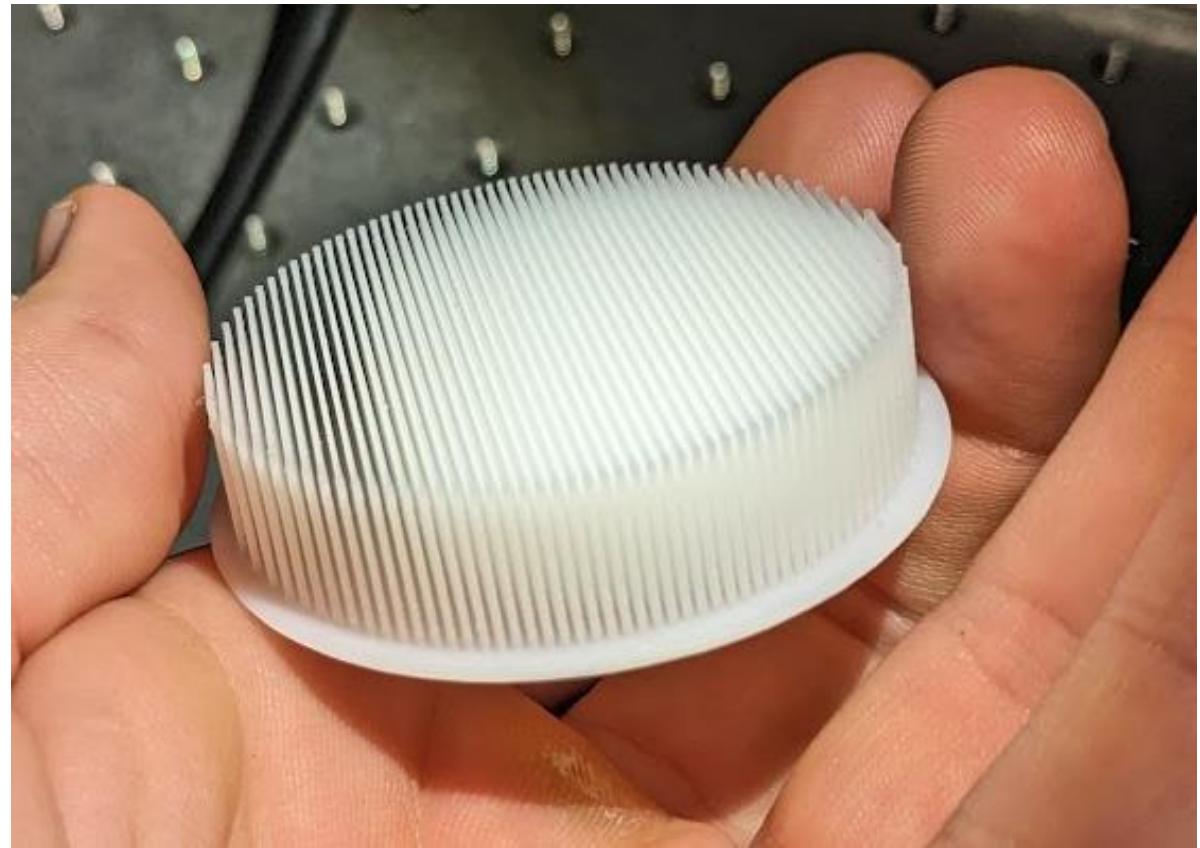
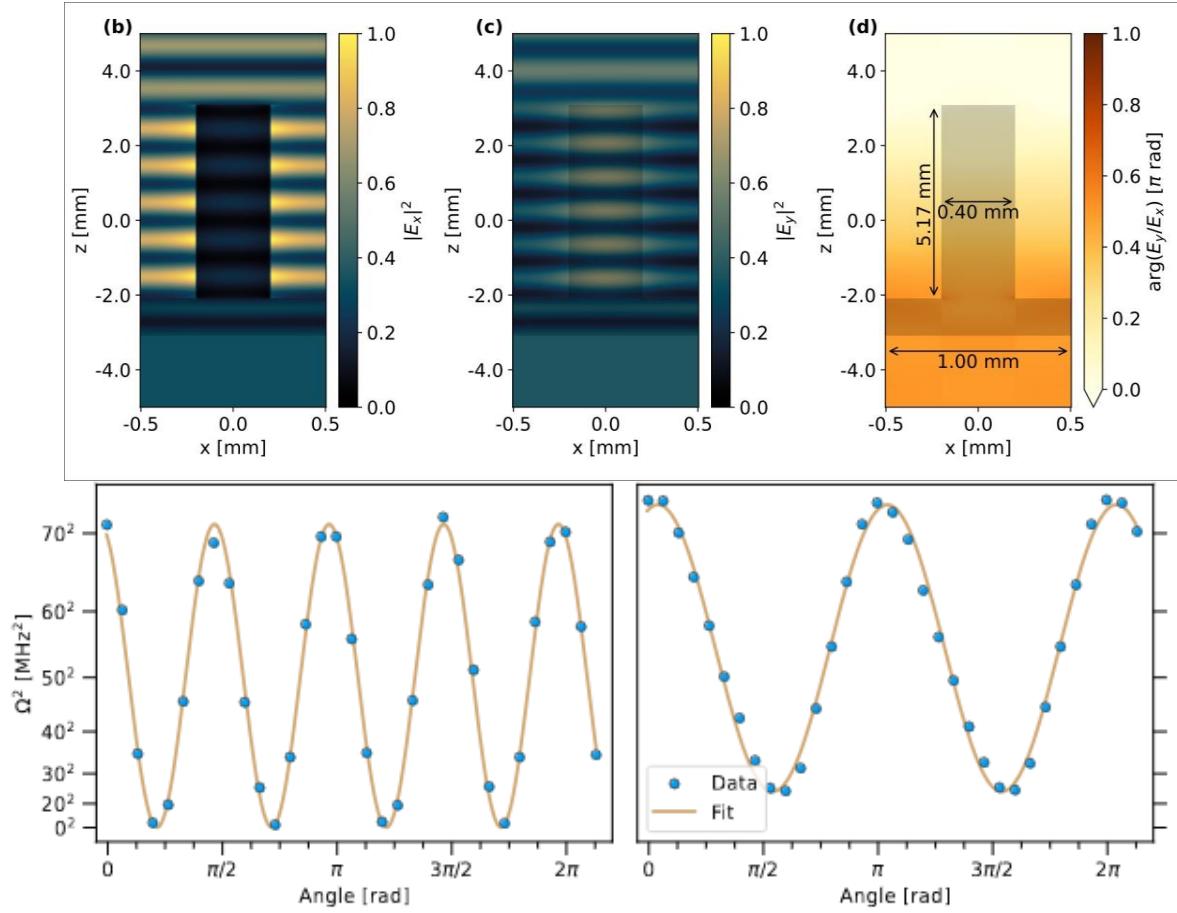
Rydberg transition
34D to 32F



Conversion @130 GHz – THz "optics" and "photonics"

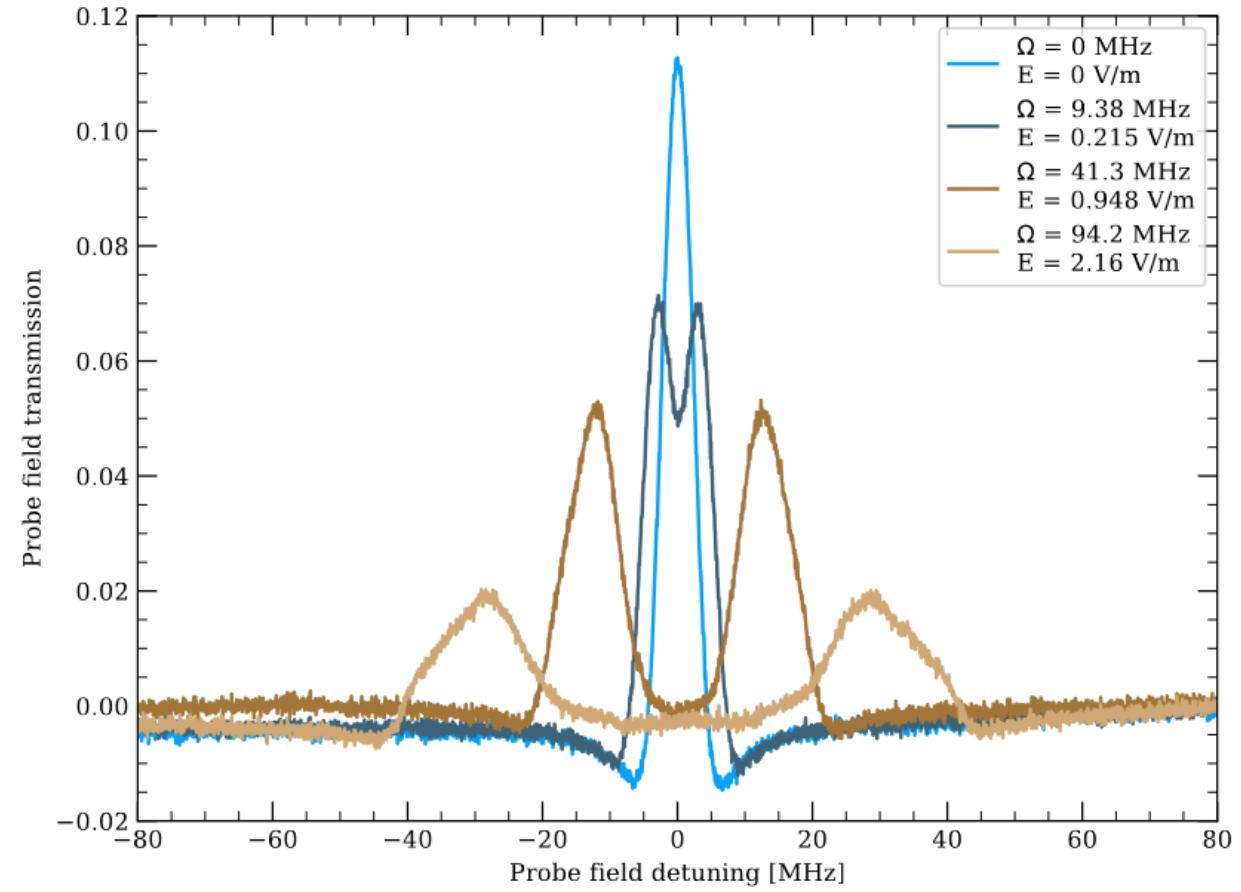
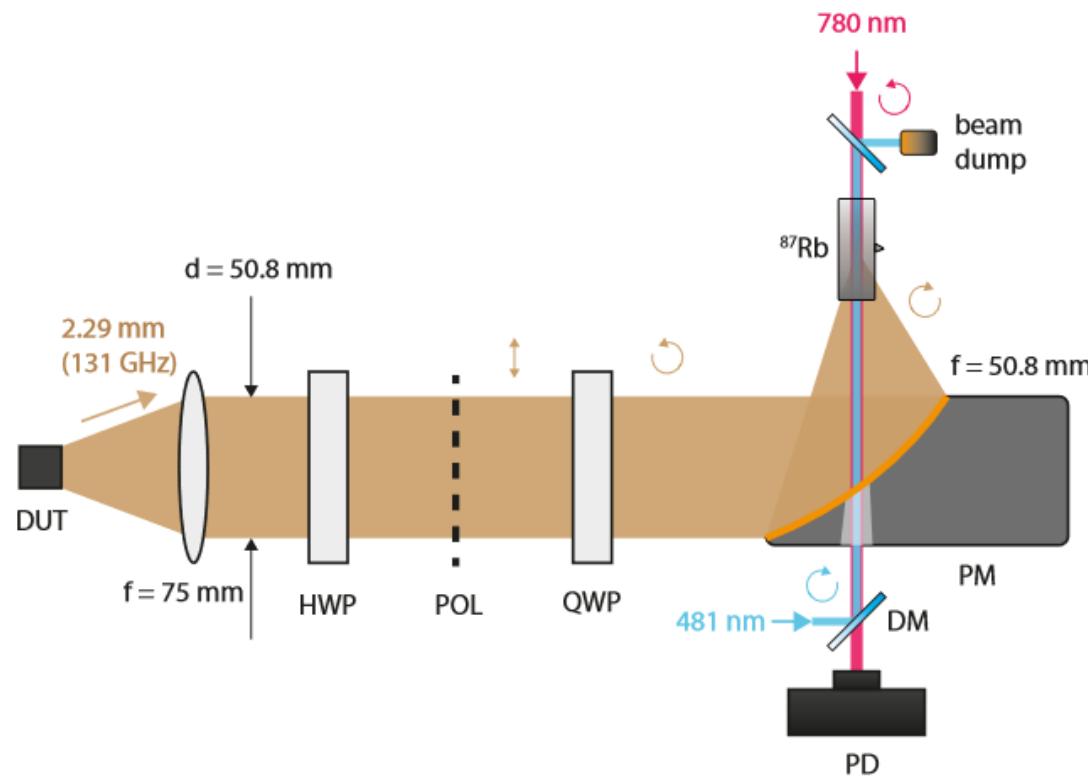


THz "photonics"

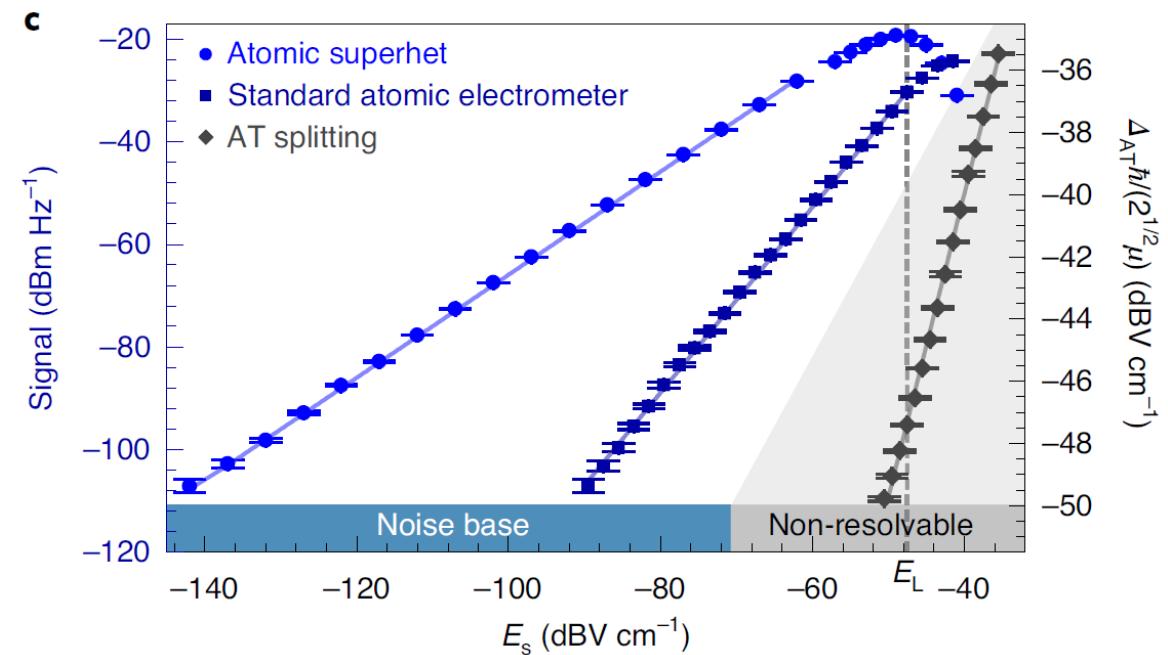
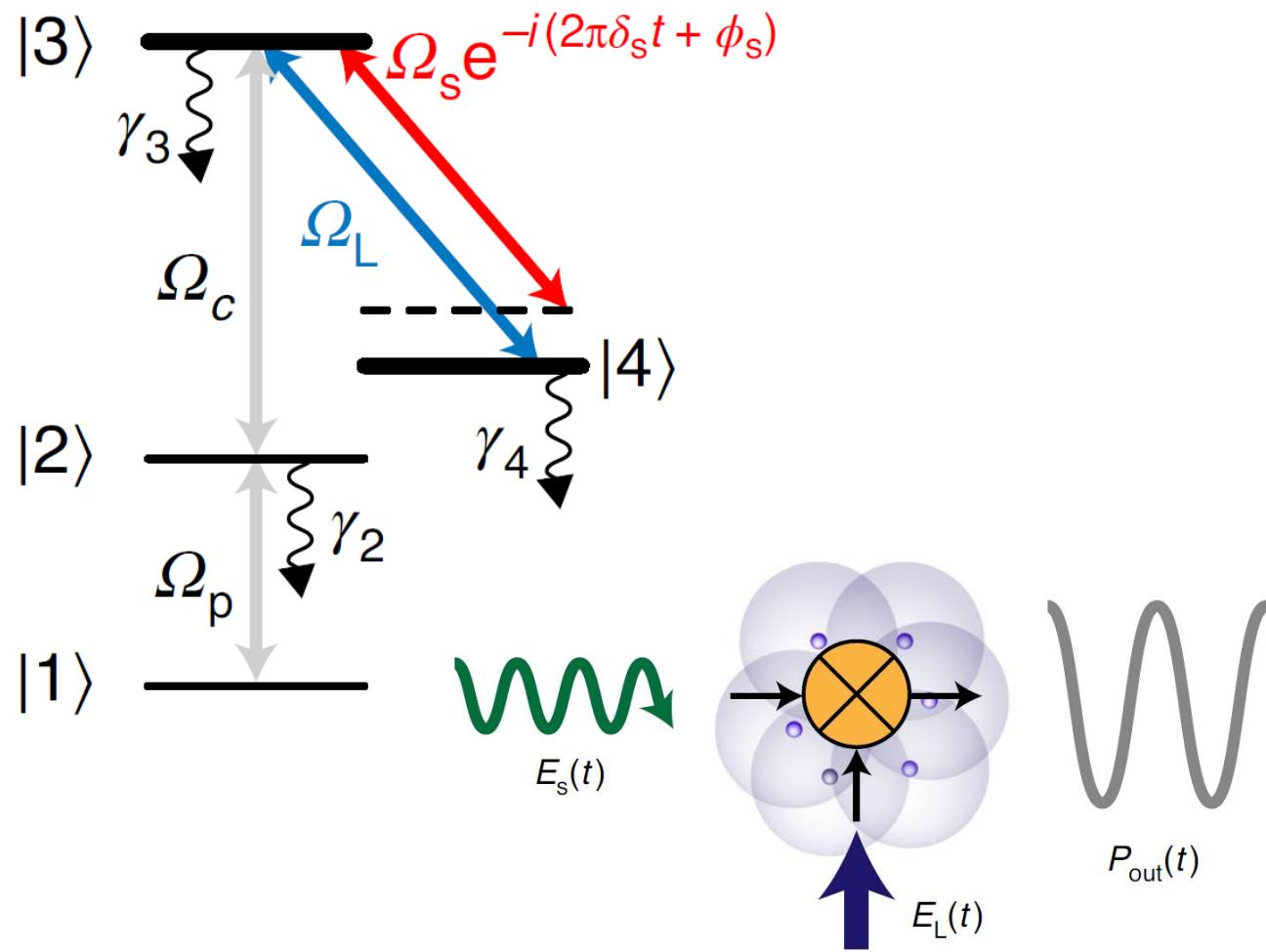


Computed using <https://github.com/flexcompute/tidy3d>

Calibration of a chip source for automotive applications (TRA120)

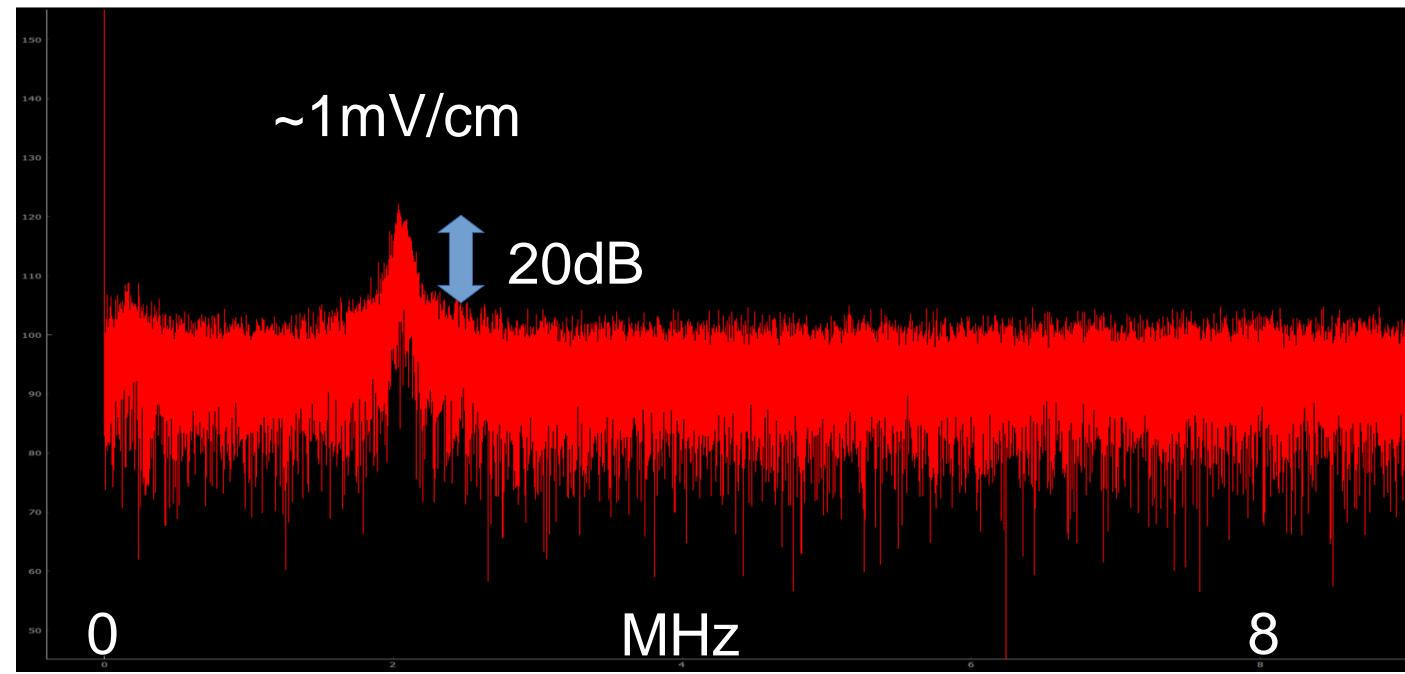
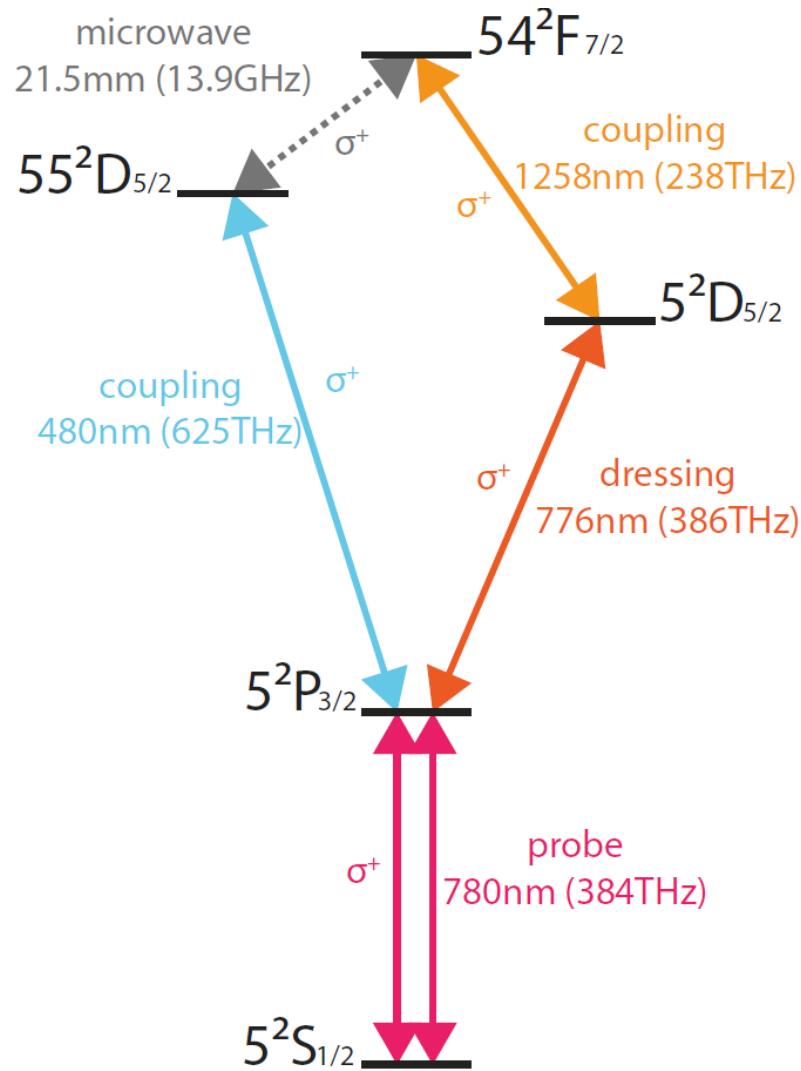


Phase sensitive detection – microwave mixer



M. Jing et al. *Nat. Phys.* **16**, 911–915 (2020)

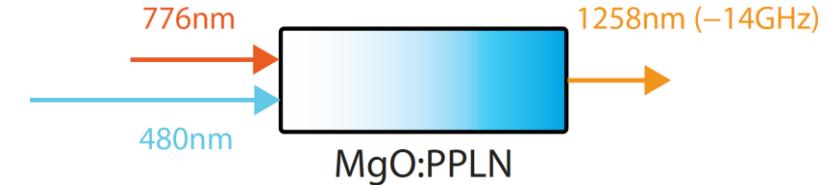
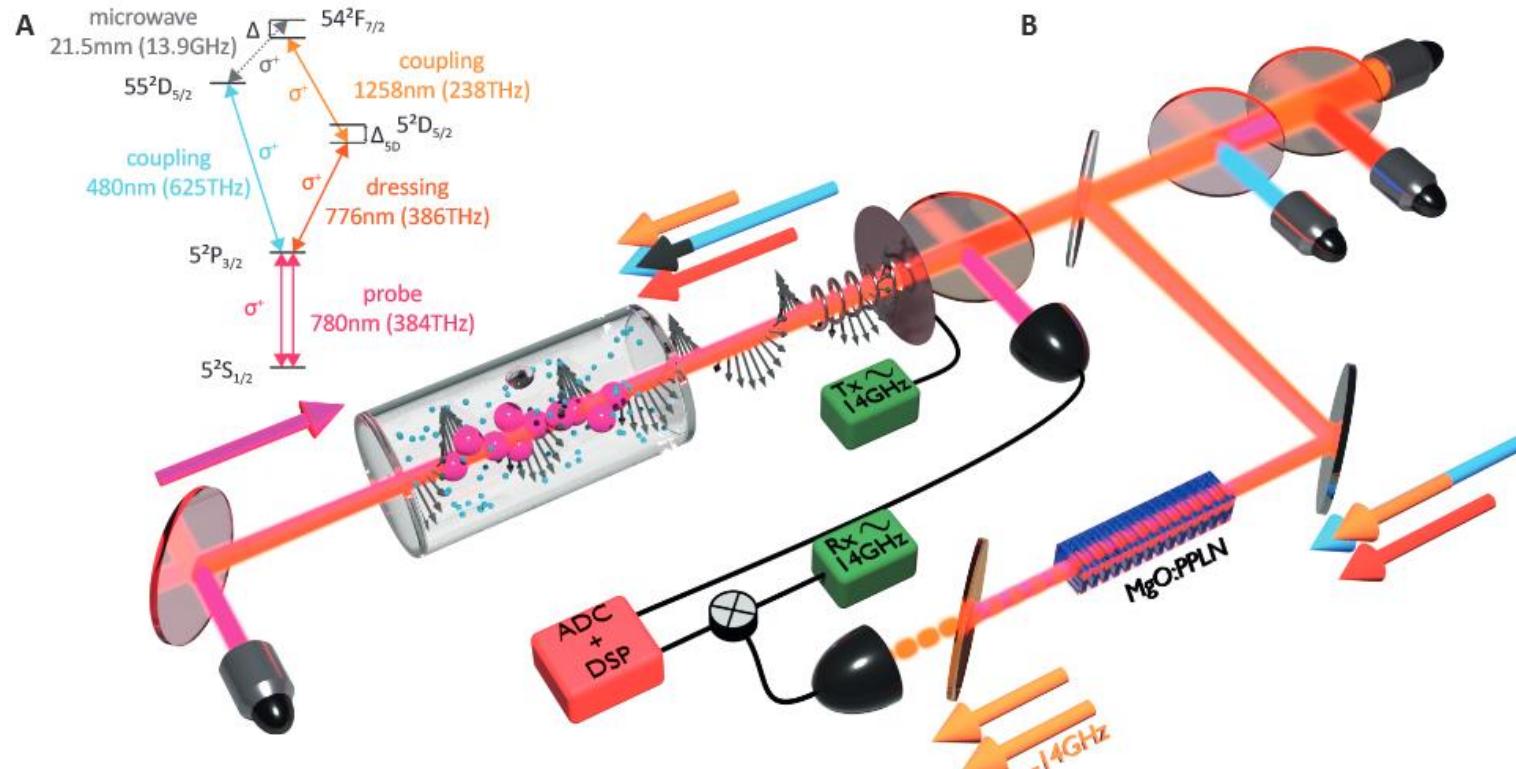
All optical superheterodyne – optically biased detection



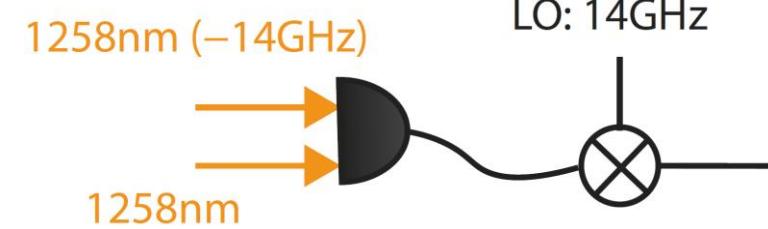
$$\phi = \phi_{480} + \phi_{776} + \phi_{1258}$$

Laser phase fluctuations transferred to the signal :(
Can we solve it without investing in ultra-stable lasers?

Noise-compensated detection

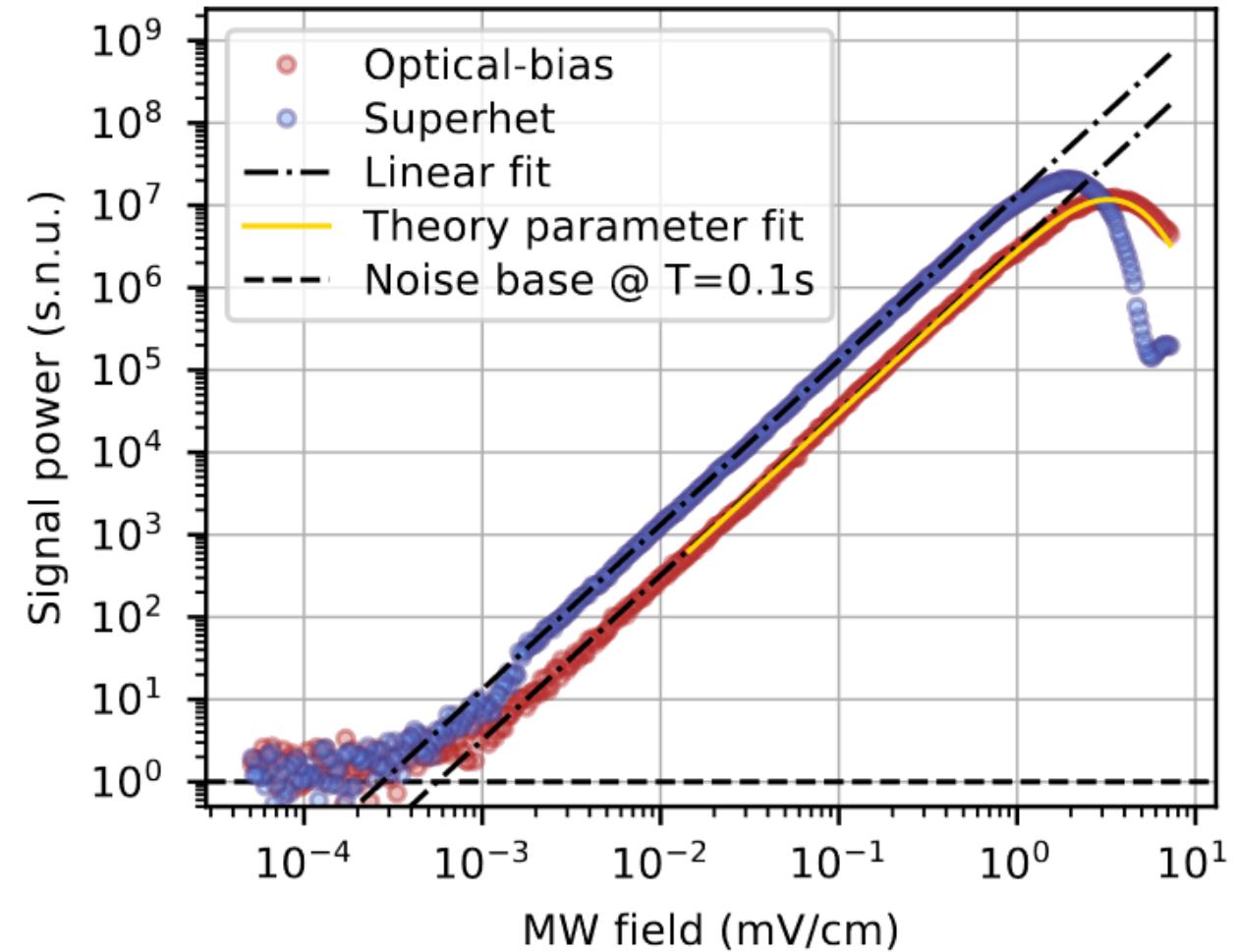
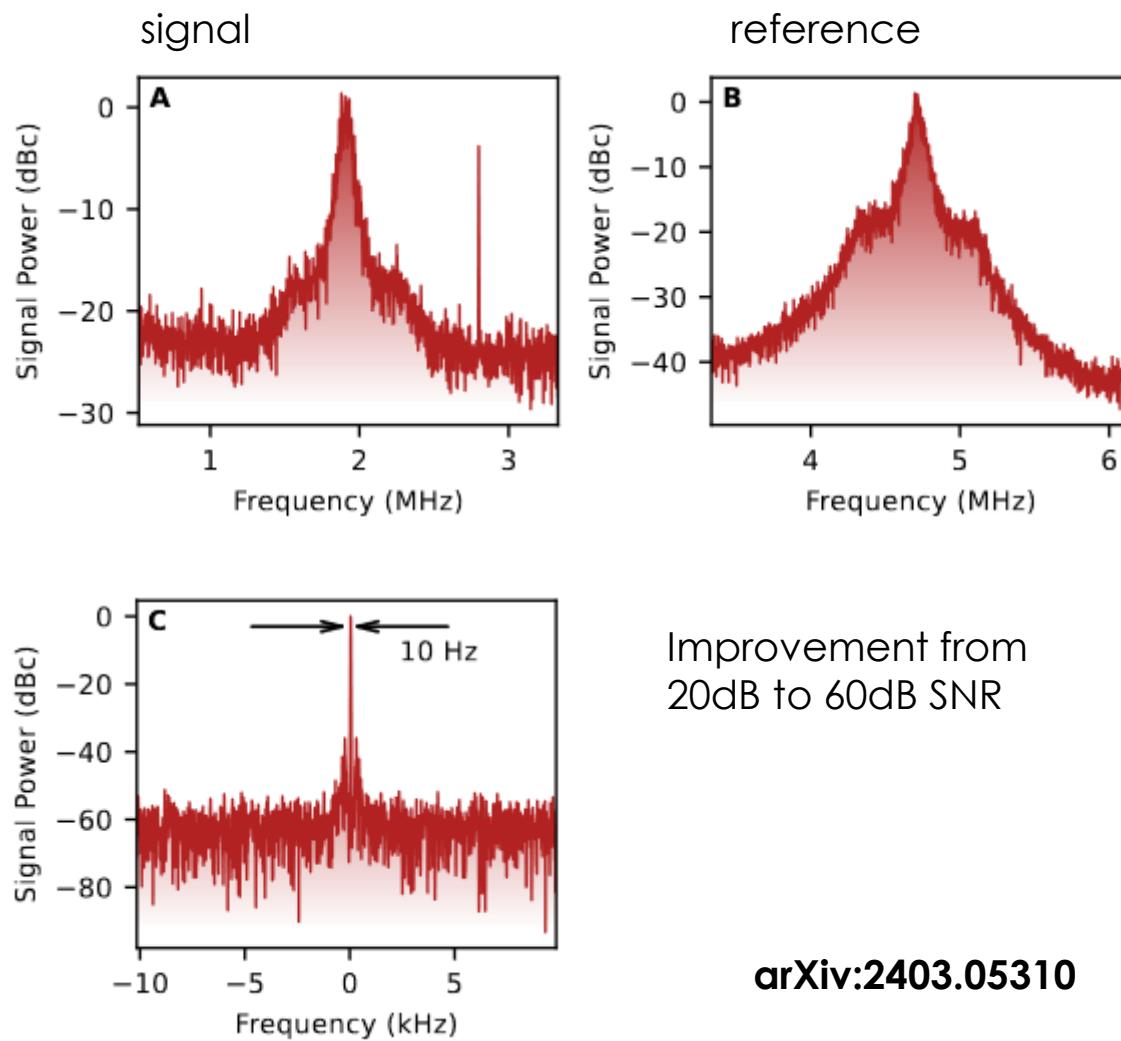


$$\phi_{480} + \phi_{776}$$



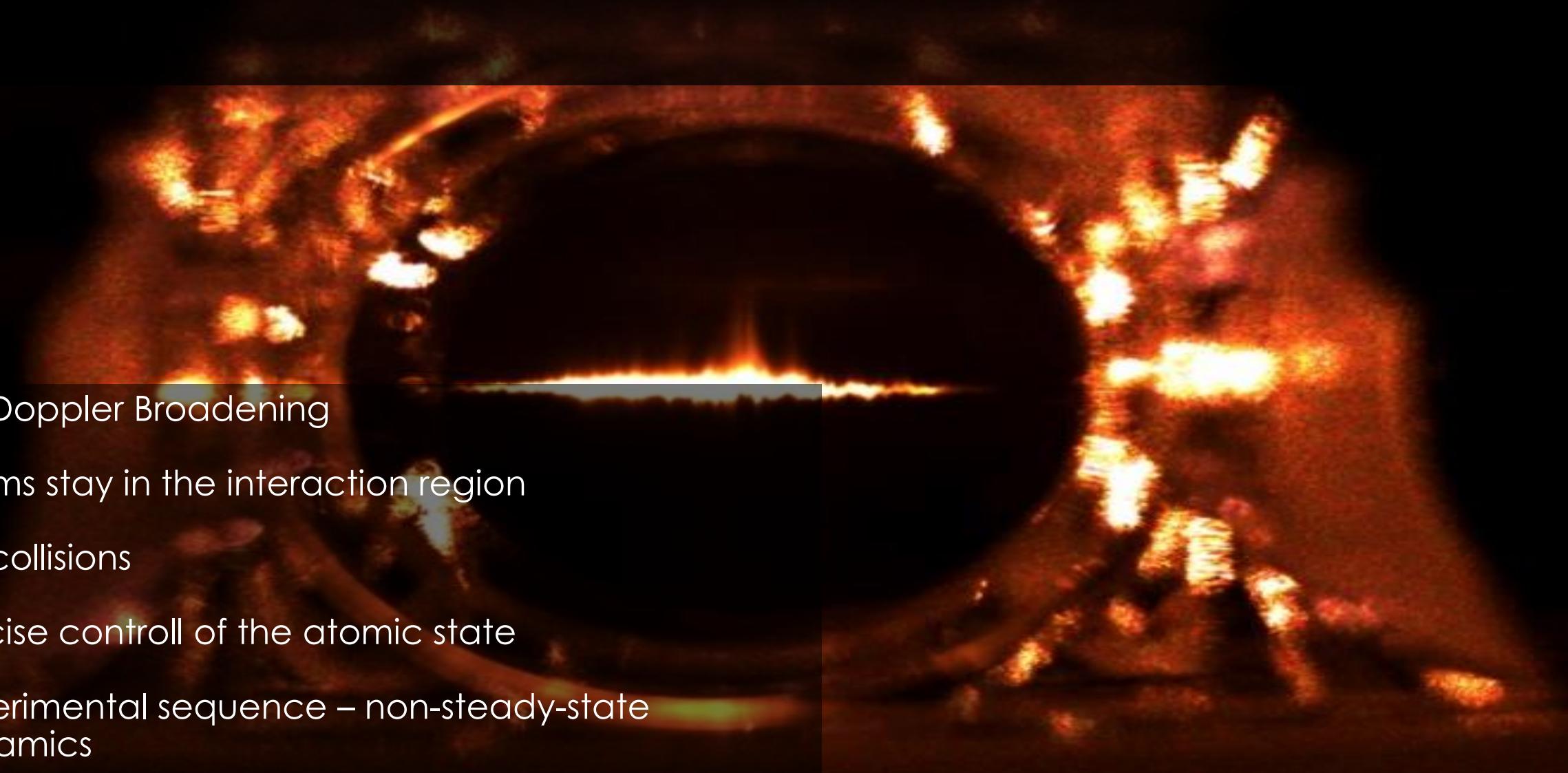
$$\phi_{480} + \phi_{776} + \phi_{1258}$$

Phase compensation and sensitivity comparison

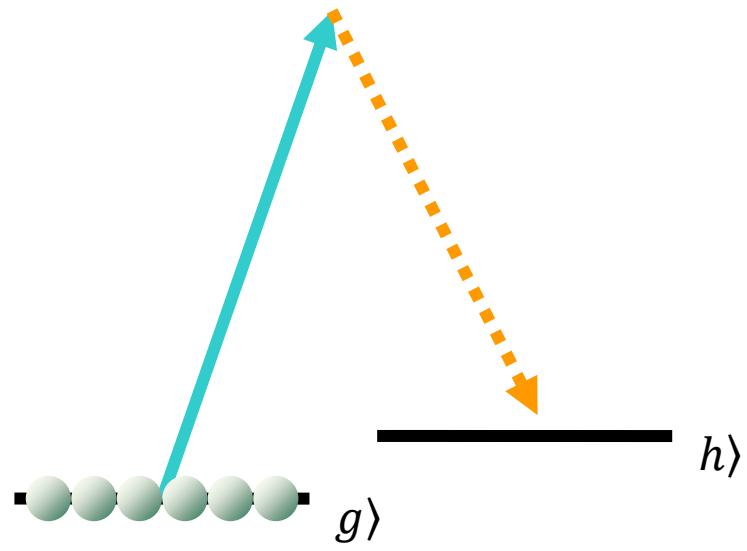


Microwave sensing with cold Rydberg atoms

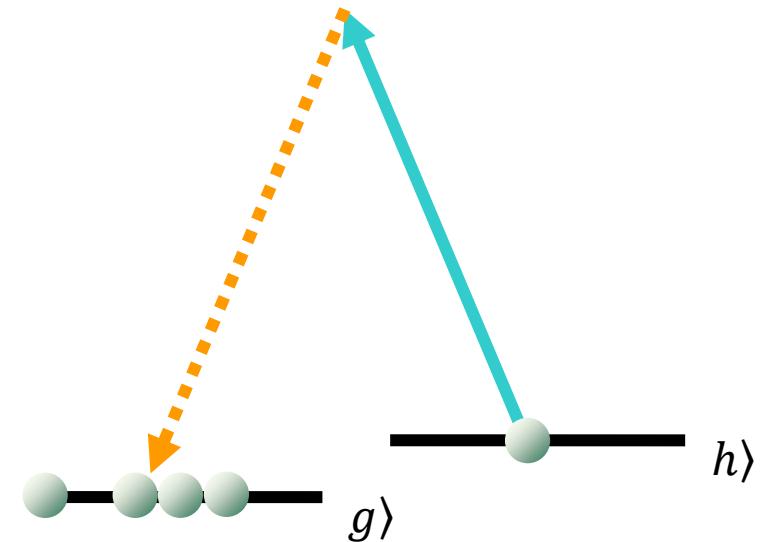
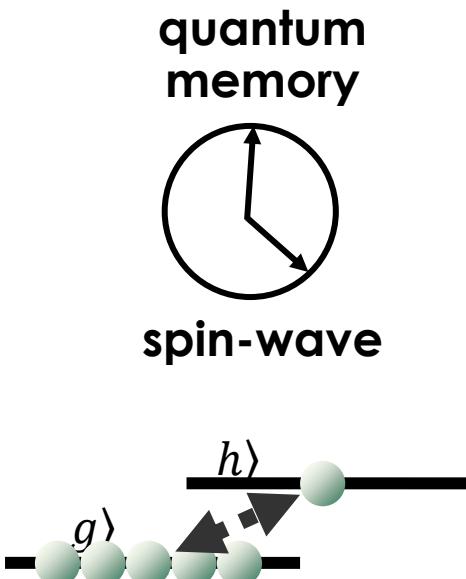
- ❑ No Doppler Broadening
- ❑ Atoms stay in the interaction region
- ❑ No collisions
- ❑ Precise control of the atomic state
- ❑ Experimental sequence – non-steady-state dynamics



Collective excitations in the ground state - interface

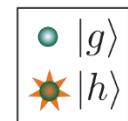


two-mode squeezed state
creation via off-resonant
Raman scattering

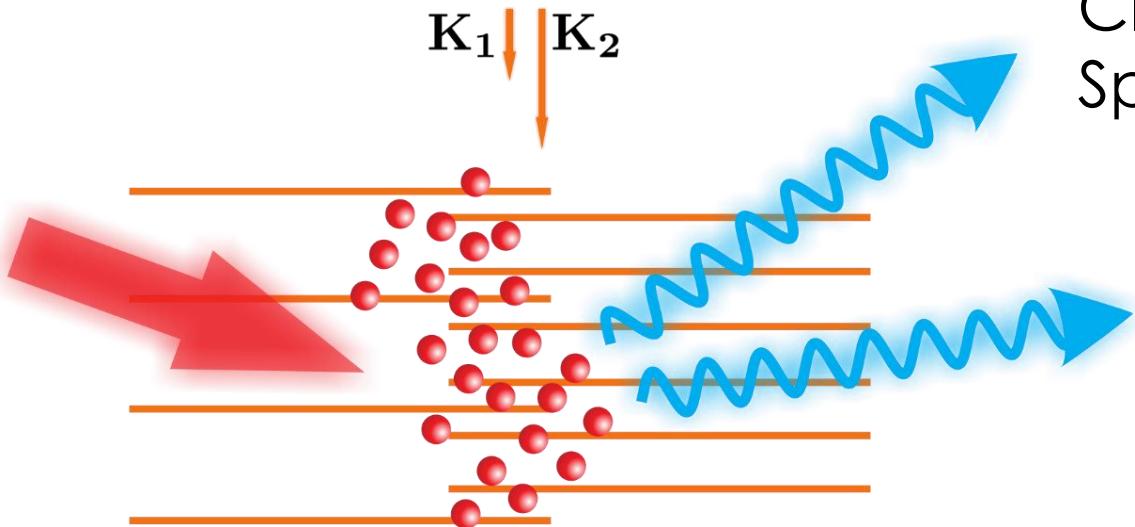


readout stage after storage time
results in **annihilation of spin-wave**

$$\frac{1}{\sqrt{N}} \left(e^{i\mathbf{K} \cdot \mathbf{r}_1} \left| \begin{array}{c} \star \\ \bullet \end{array} \right. \right) + e^{i\mathbf{K} \cdot \mathbf{r}_2} \left| \begin{array}{c} \star \\ \bullet \end{array} \right. + e^{i\mathbf{K} \cdot \mathbf{r}_3} \left| \begin{array}{c} \star \\ \bullet \end{array} \right. + \dots$$



Spin-wave storage and readout



Classical analogue
Spatially-dependent atomic coherence

$$\hat{\rho}(\mathbf{r}) = \frac{1}{1 + |\beta(\mathbf{r})|^2} \begin{pmatrix} 1 & \beta(\mathbf{r})e^{i\mathbf{K}\cdot\mathbf{r}} \\ \beta^*(\mathbf{r})e^{-i\mathbf{K}\cdot\mathbf{r}} & |\beta(\mathbf{r})|^2 \end{pmatrix}.$$

$$\tilde{\mathbf{P}} = n \text{Tr}(\rho \hat{\mathbf{d}}),$$

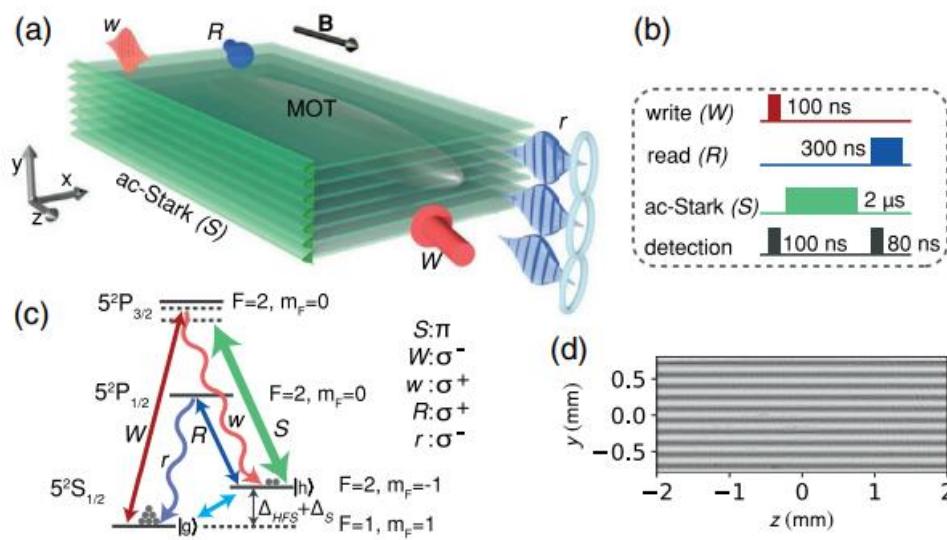
$$2i \frac{\partial \mathbf{A}}{\partial z} = -\frac{\omega}{c\epsilon_0} \mathbf{P}.$$

$$\frac{1}{\sqrt{N}} \left(e^{i\mathbf{K}\cdot\mathbf{r}_1} |\star\bullet\bullet\bullet\rangle + e^{i\mathbf{K}\cdot\mathbf{r}_2} |\bullet\star\bullet\bullet\rangle + e^{i\mathbf{K}\cdot\mathbf{r}_3} |\bullet\bullet\star\bullet\rangle + \dots \right)$$

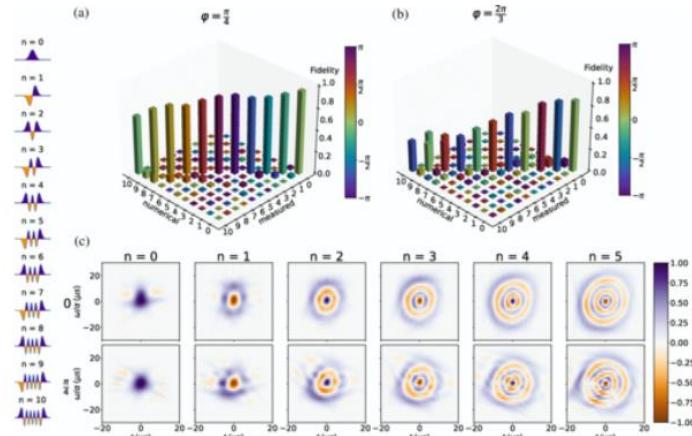
$$\tau = \left(|\mathbf{k}| \sqrt{\frac{k_B T}{m}} \right)^{-1}$$

Result: directional emission due to phase matching

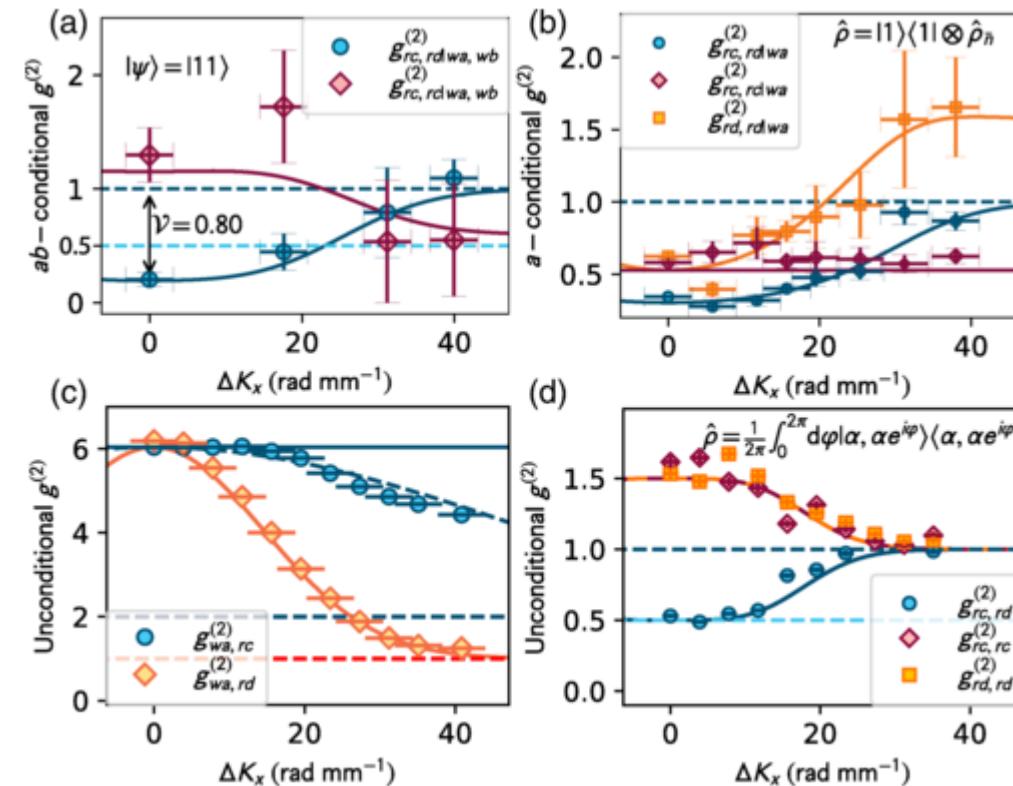
Collective excitations



M. Parniak, et al. Phys. Rev. Lett. **122**, 063604



B. Niewelt et al. Phys. Rev. Lett. **130**, 240801

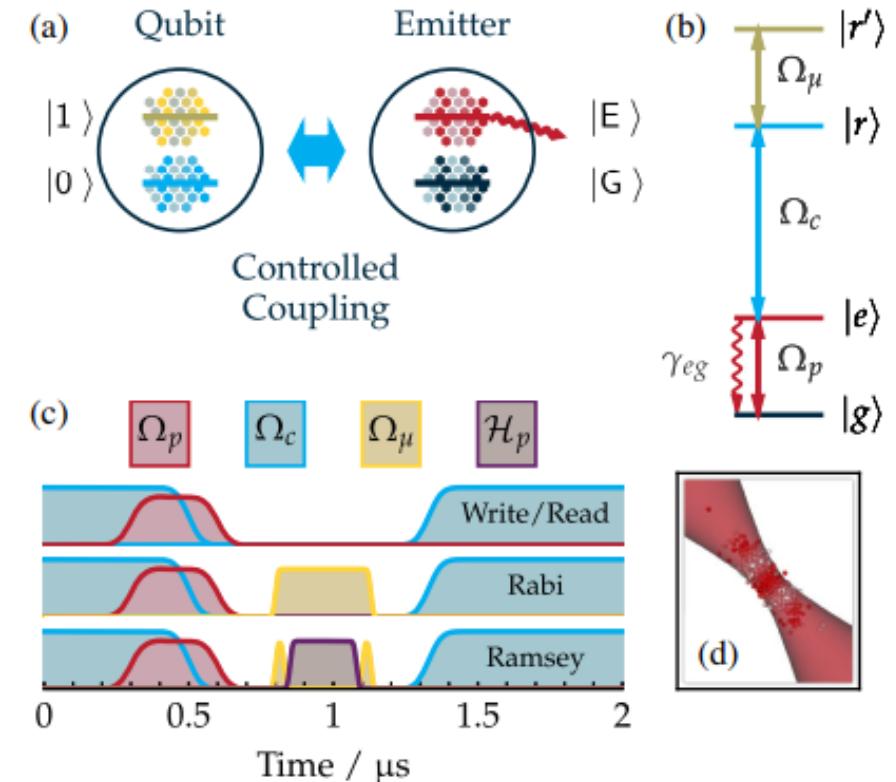


Collective Rydberg qubits

- × quantum information is distributed over many atoms
- × direct readout in a well-defined spatial mode

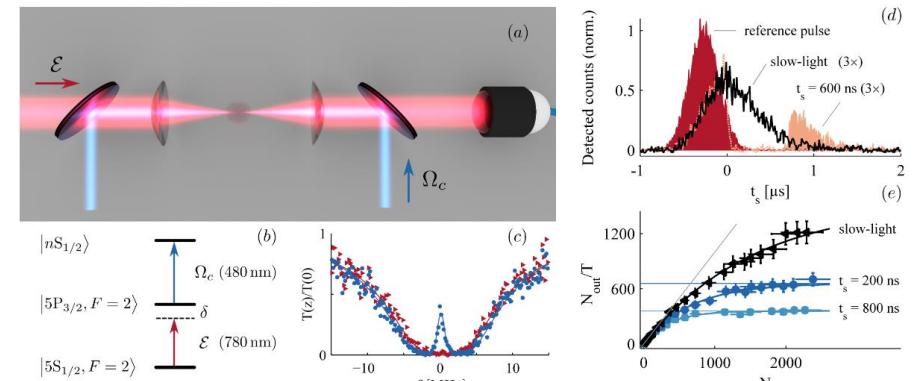
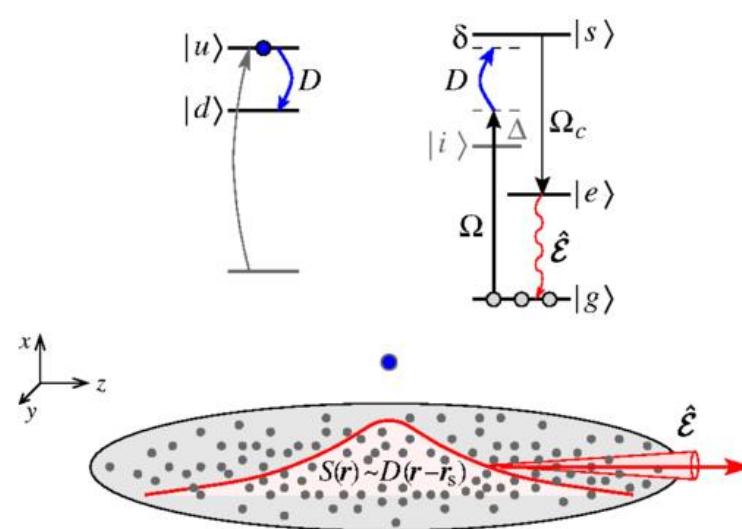
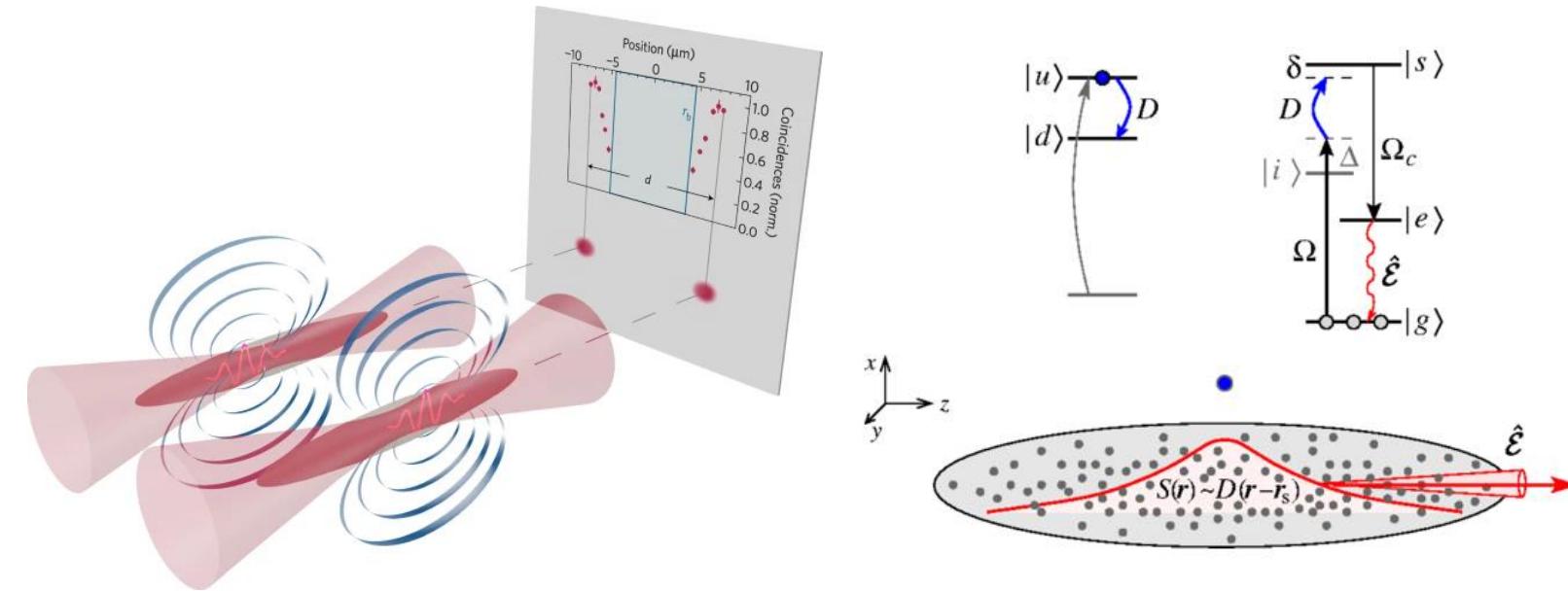
$$|0\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{i(\mathbf{k} \cdot \mathbf{R}_j - \omega_r t)} |g_1, g_2, \dots, r_j, \dots, g_N\rangle$$

$$|1\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{i(\mathbf{k} \cdot \mathbf{R}_j - \omega_{r'} t)} |g_1, g_2, \dots, r'_j, \dots, g_N\rangle$$



Collectively Encoded Rydberg Qubit
Nicholas L. R. Spong, et al., Phys. Rev. Lett. **127**, 063604

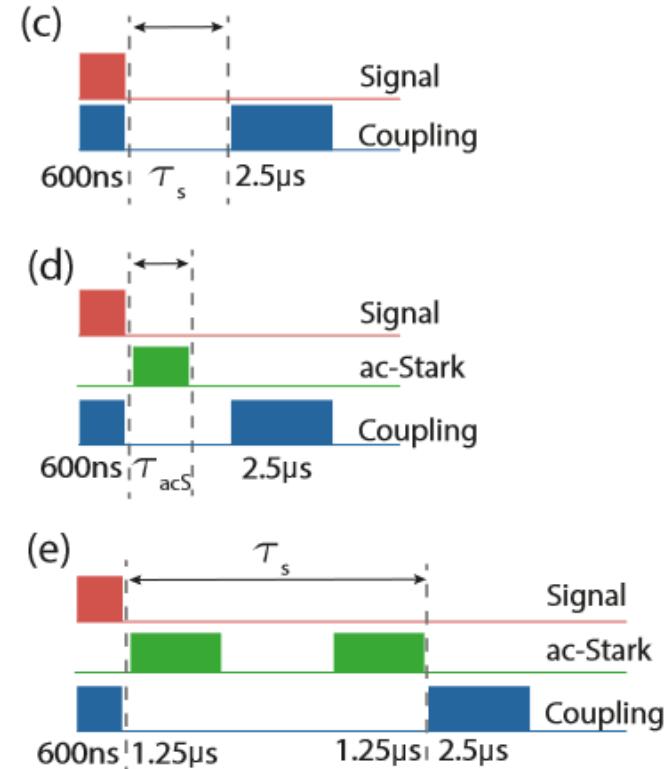
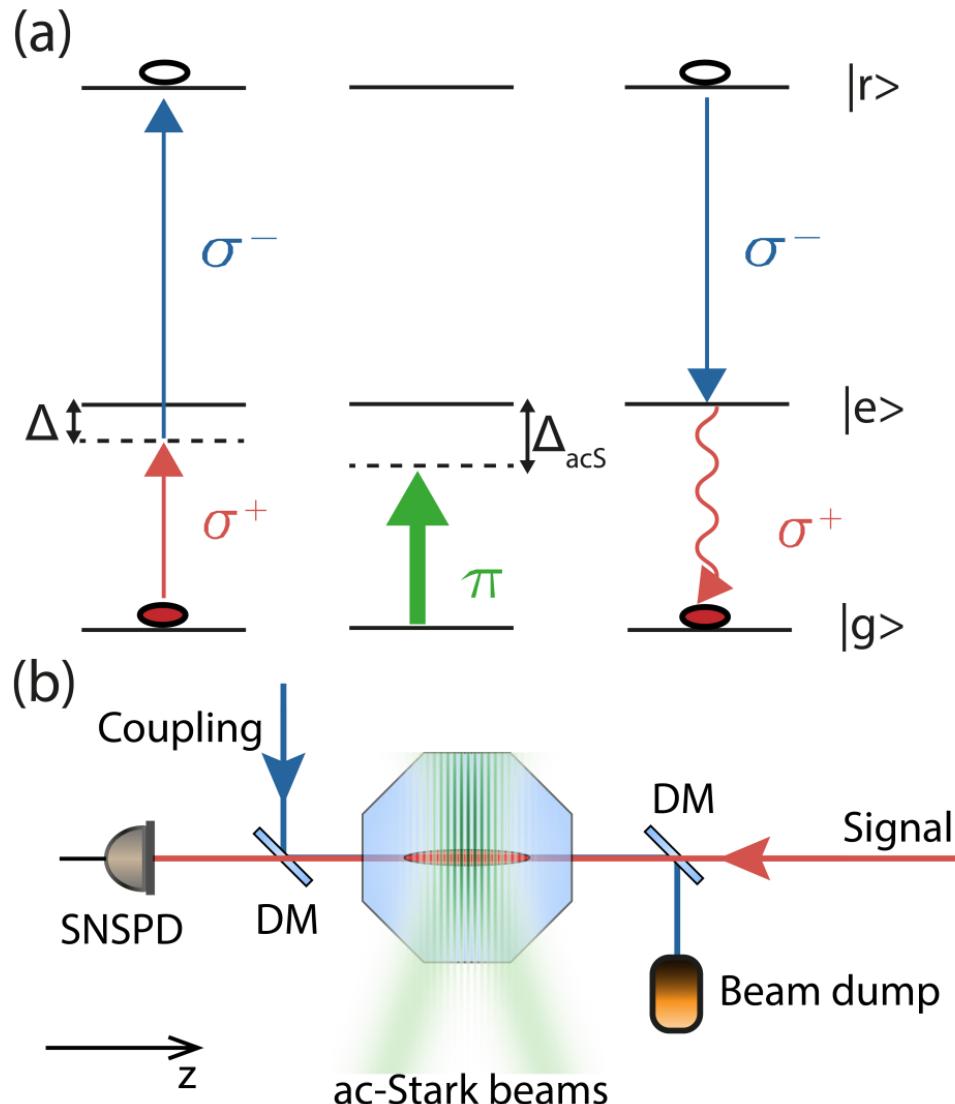
Collective Rydberg qubits



E. Distante et al., Phys. Rev. Lett. 117, 113001 (2016)

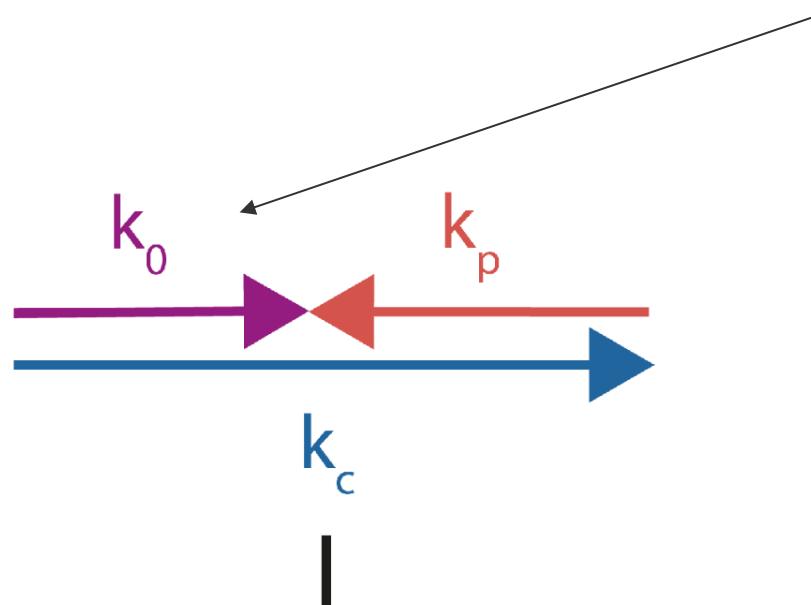
H. Busche, et al., Nature Physics 13, 655-658 (2017) D. Petrosyan and K. Mølmer, Phys. Rev. Lett. 121, 123605 (2021)

EIT preparation and retrieval

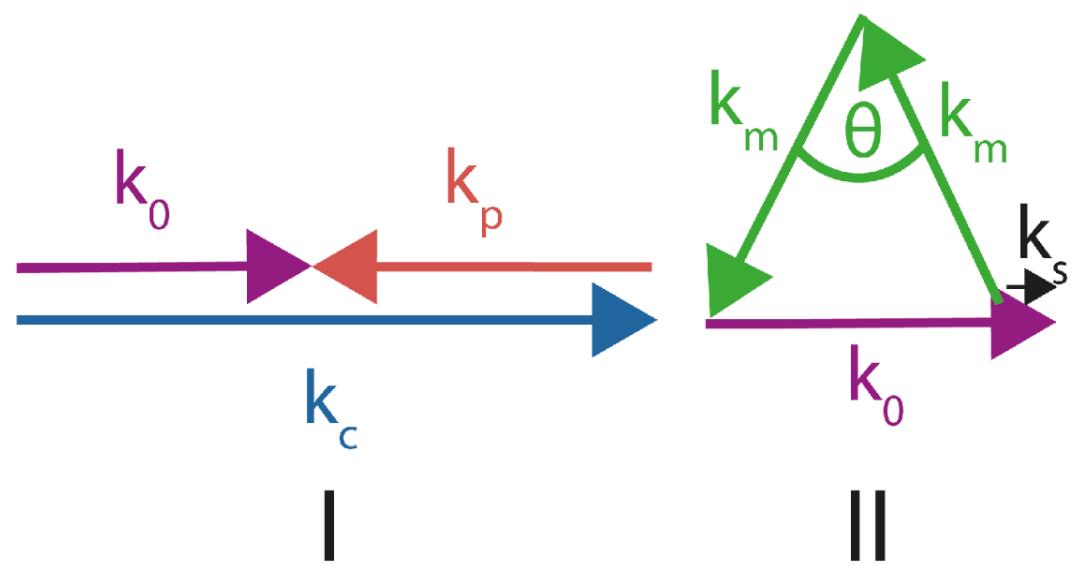


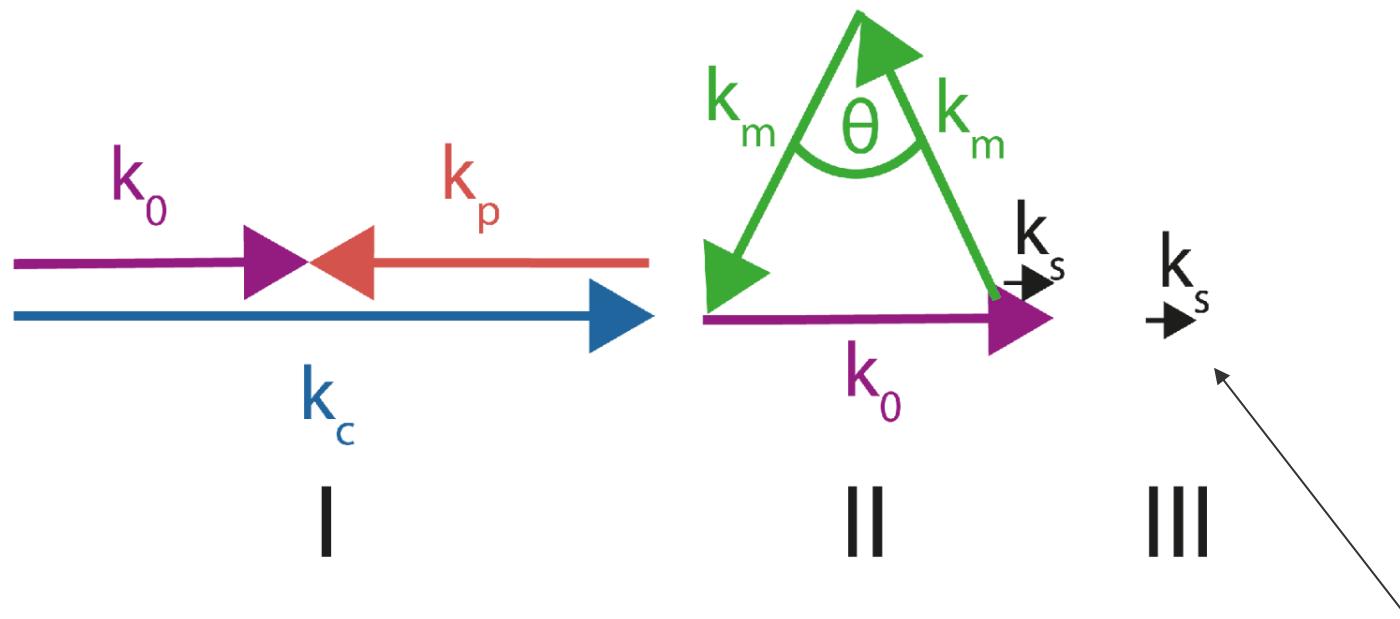
S. Kurzyna, B. Niewelt, W. Wasilewski, M. Mazelanik, M. Parniak
arXiv:2402:06513, Quantum 8, 1431 (2024)

Non-zero excitation wavevector ->
lifetime reduced to only 1 μ s

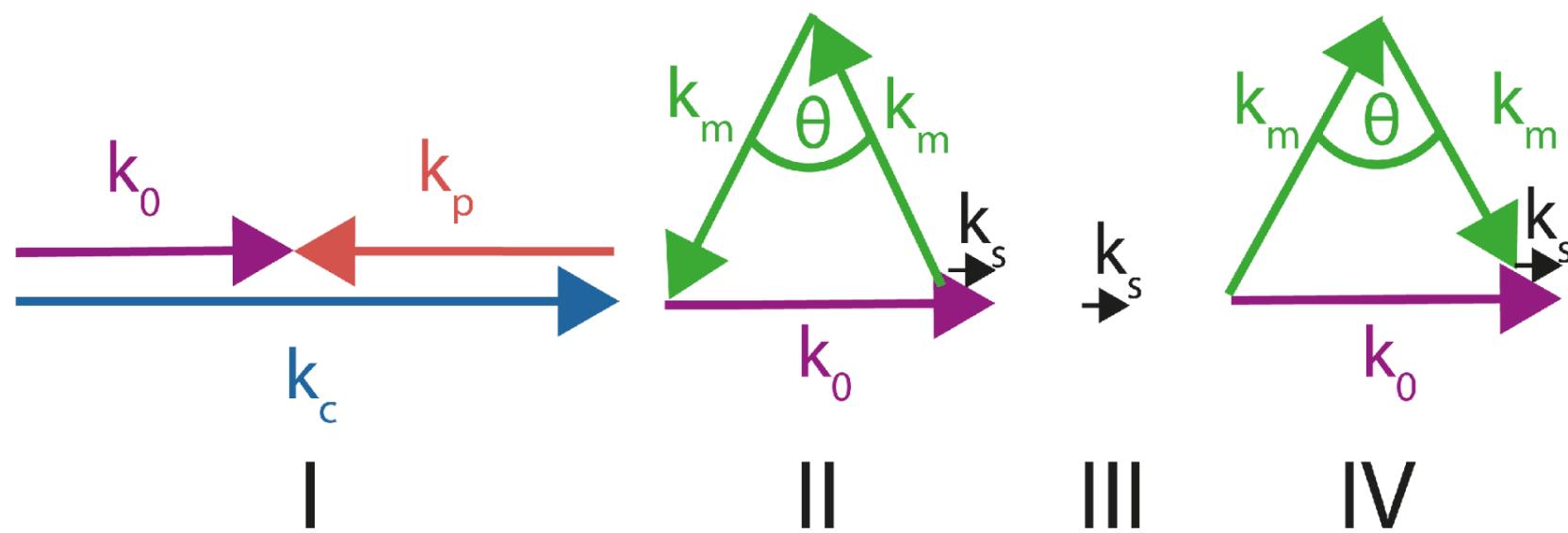


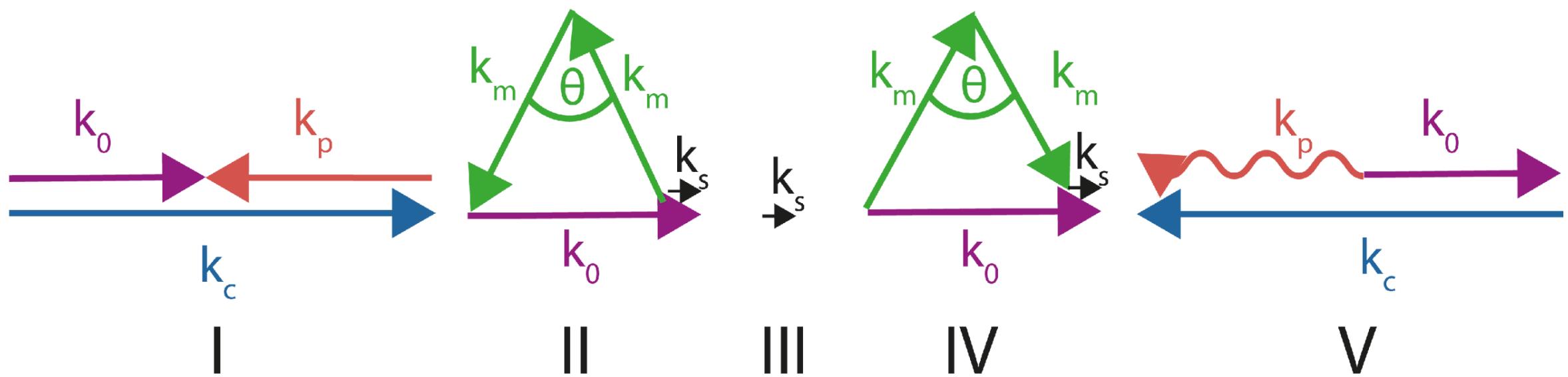
$$\tau = \left(|\mathbf{k}| \sqrt{\frac{k_B T}{m}} \right)^{-1}$$

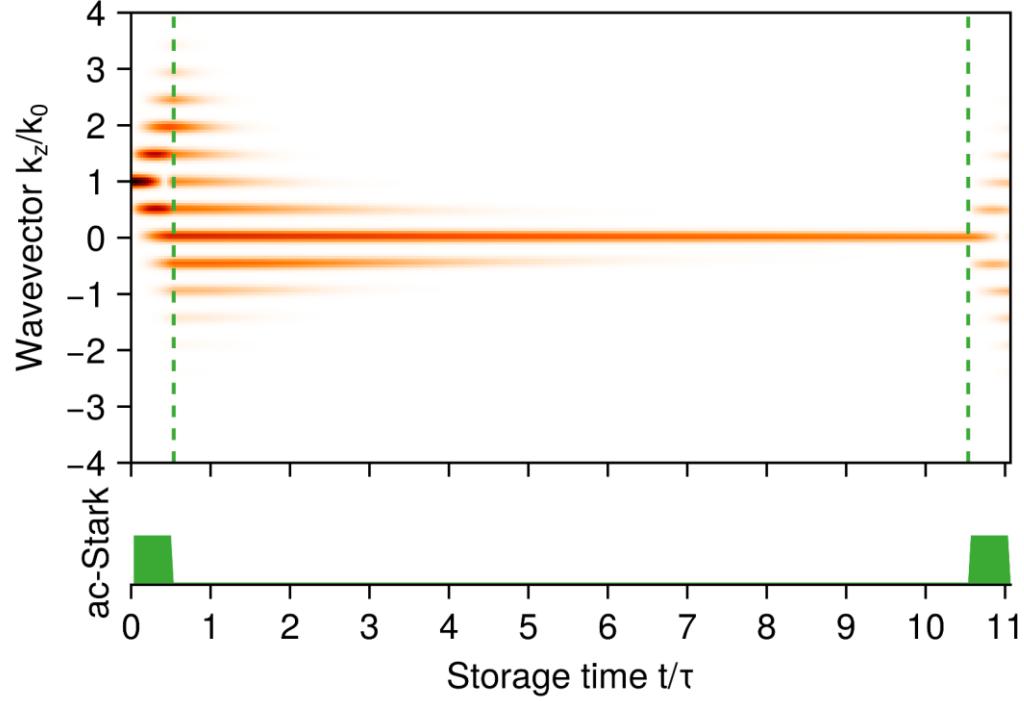




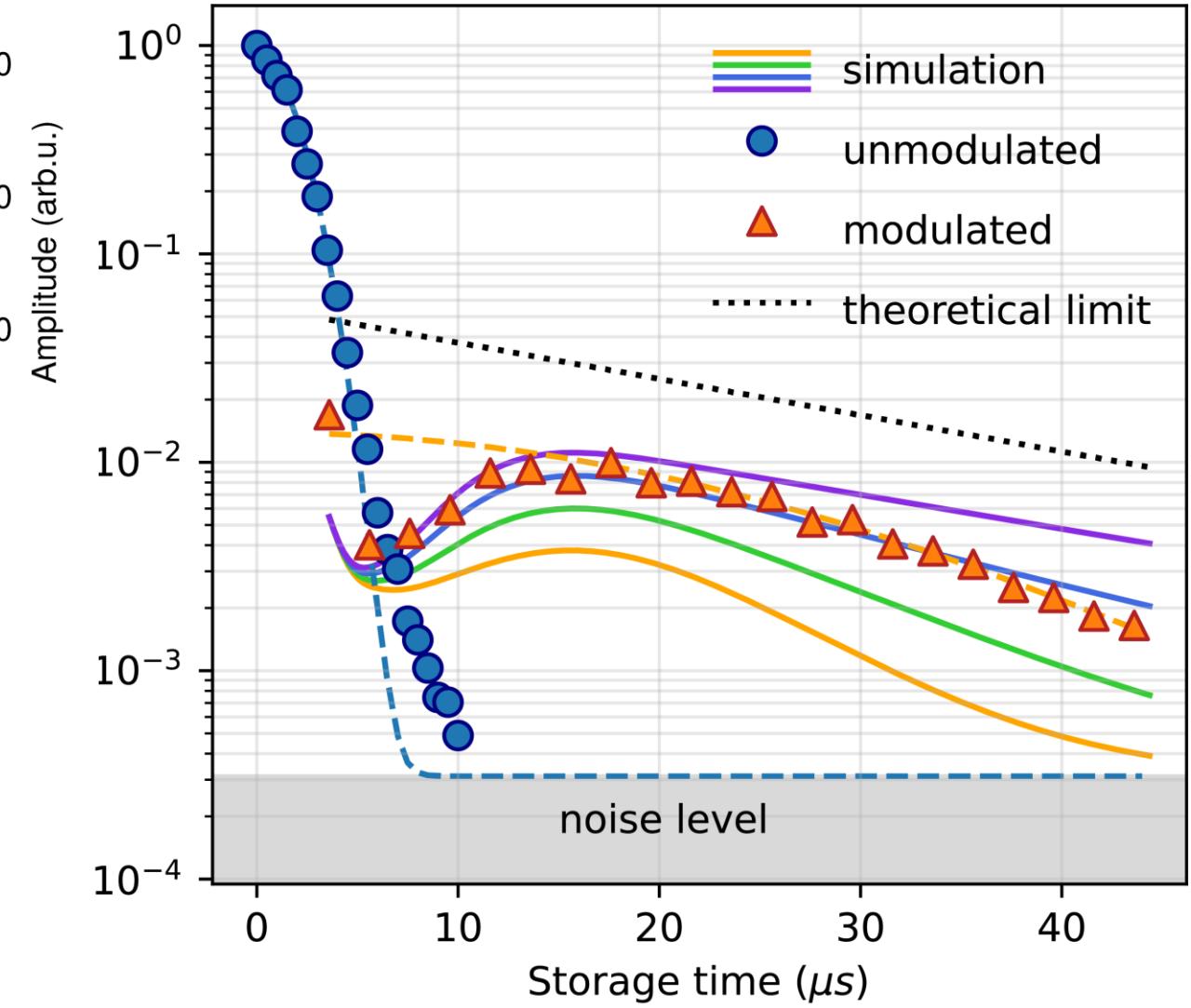
Thermal decay is halted







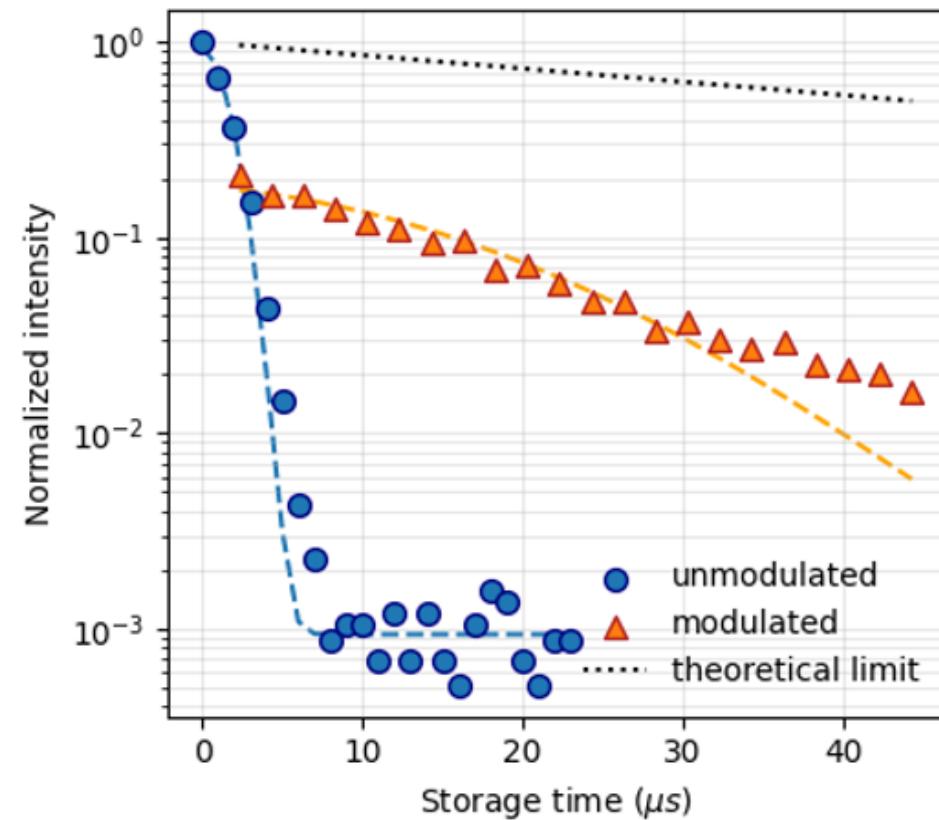
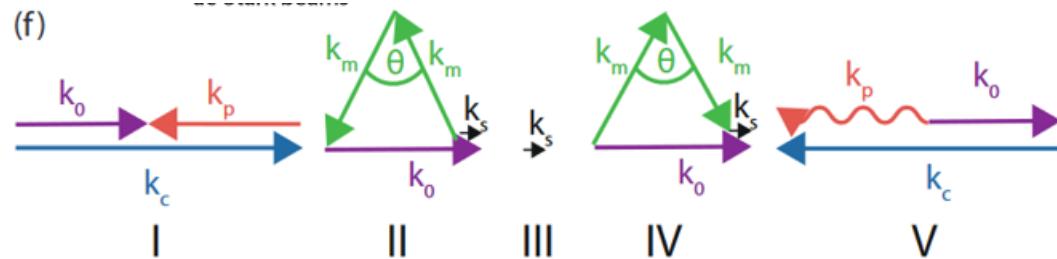
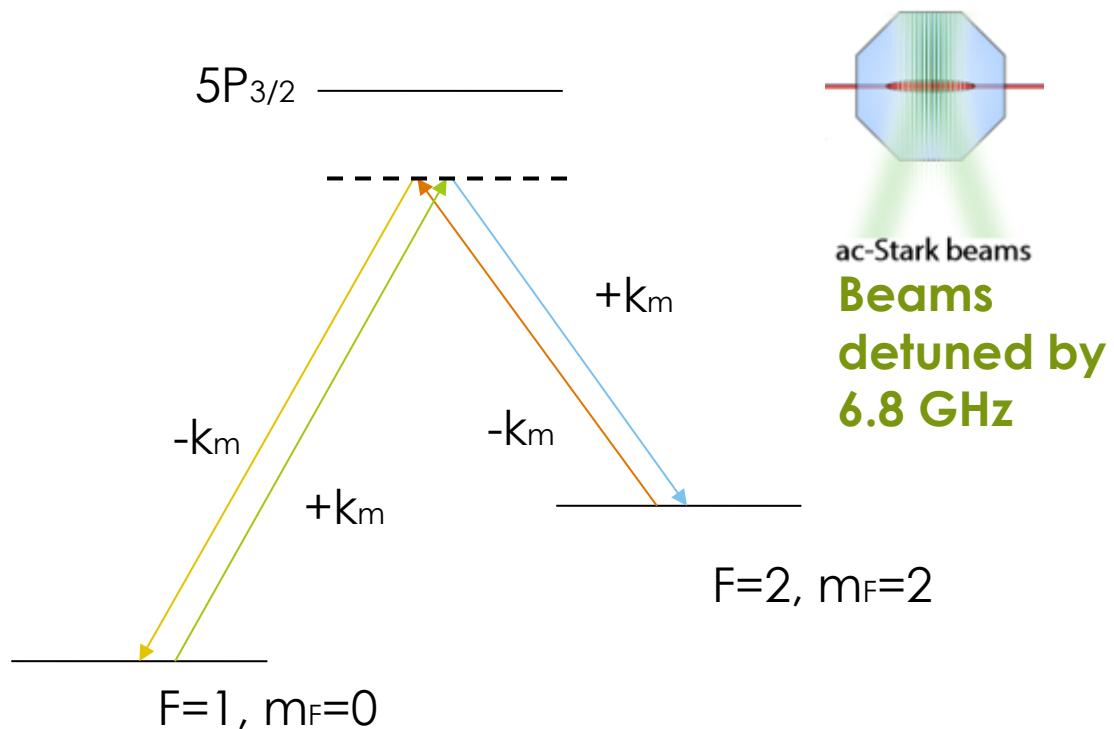
Efficiency is limited due to additional diffraction orders $\sim 5.4\%$, yet this still gives large improvement for longer storage times



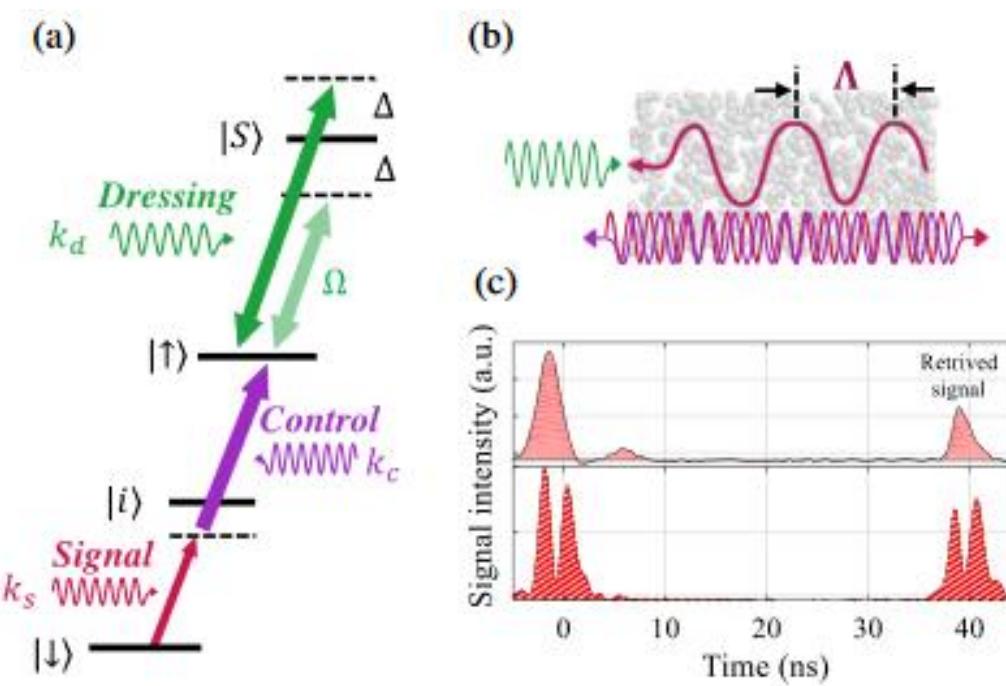
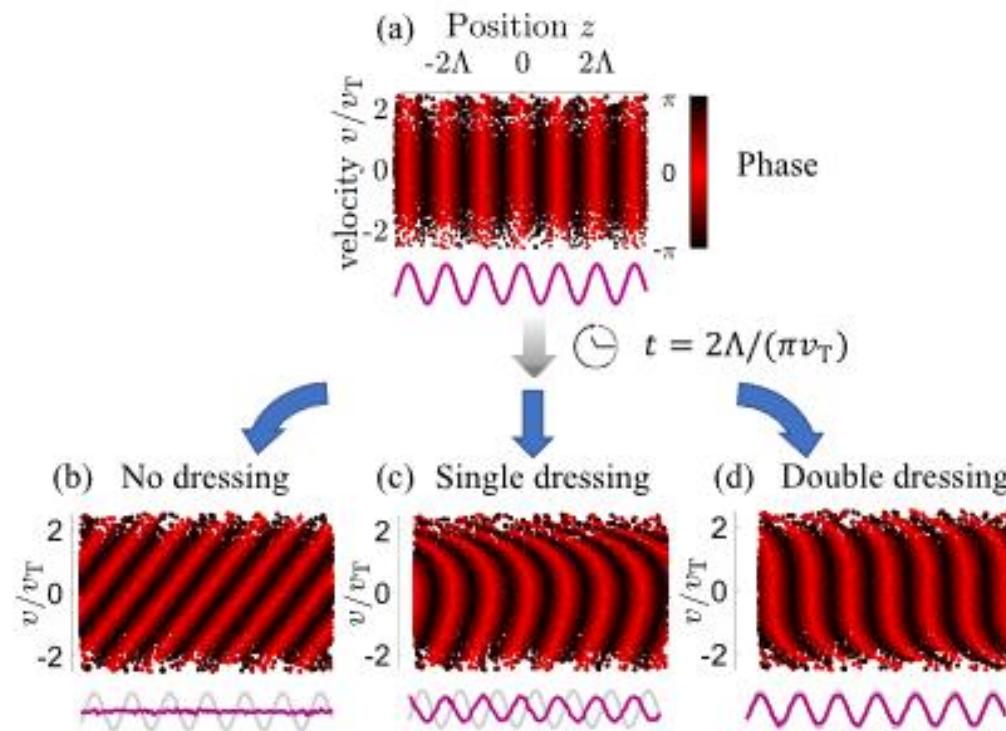
S. Kurzyna, B. Niewelt, W. Wasilewski, M. Mazelanik, M. Parniak
Quantum 8, 1431 (2024)

State-selective modulation

Solution: two-Photon Rabi oscillation/Raman transition between hyperfine states with simultaneous momentum transfer

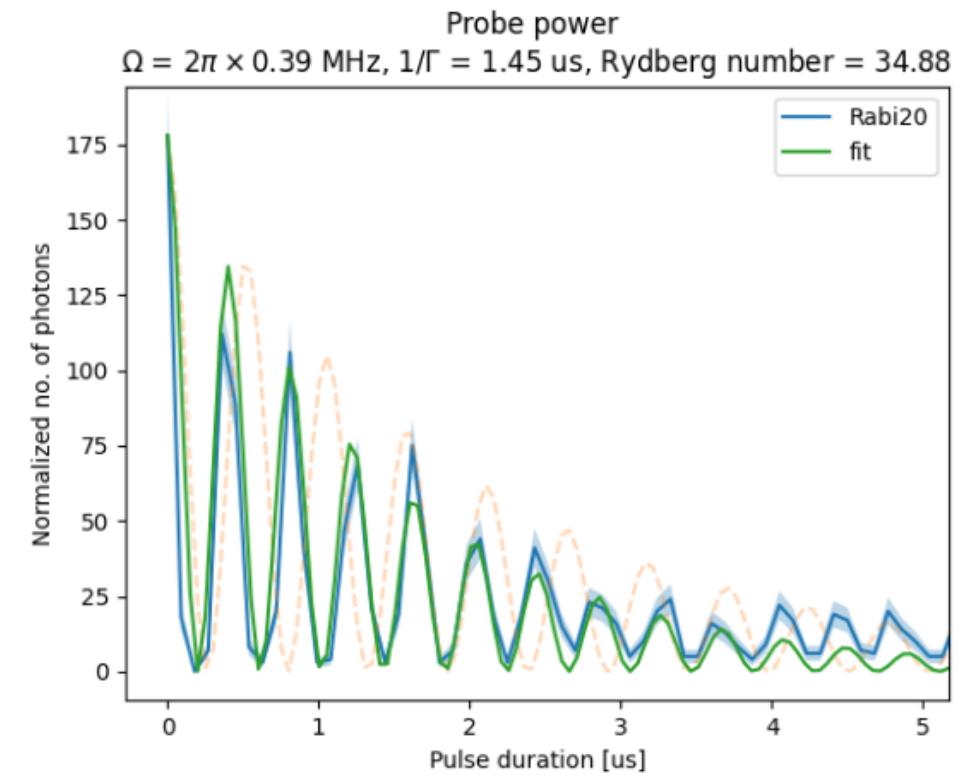
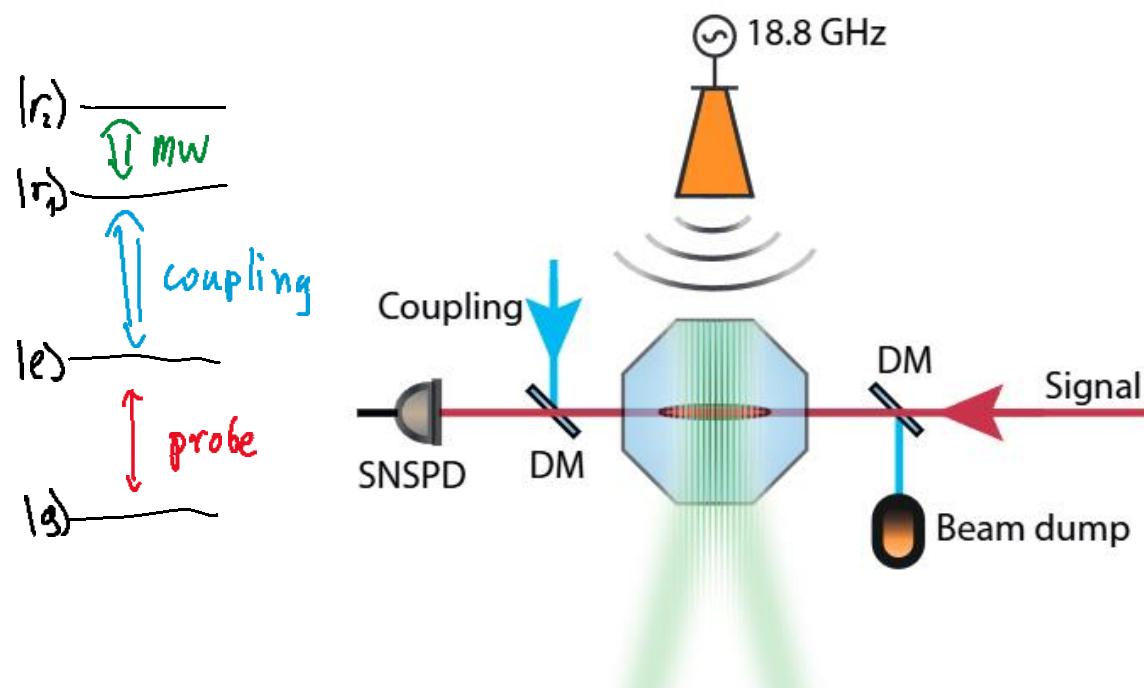


Possible also in hot atoms



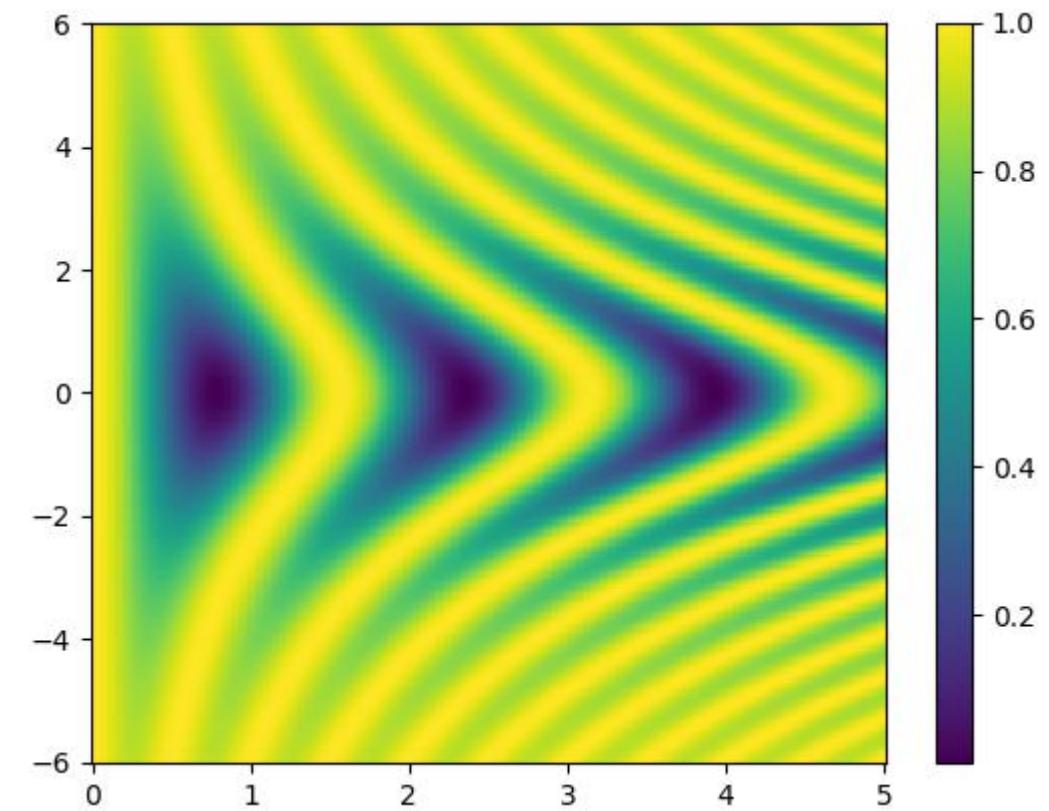
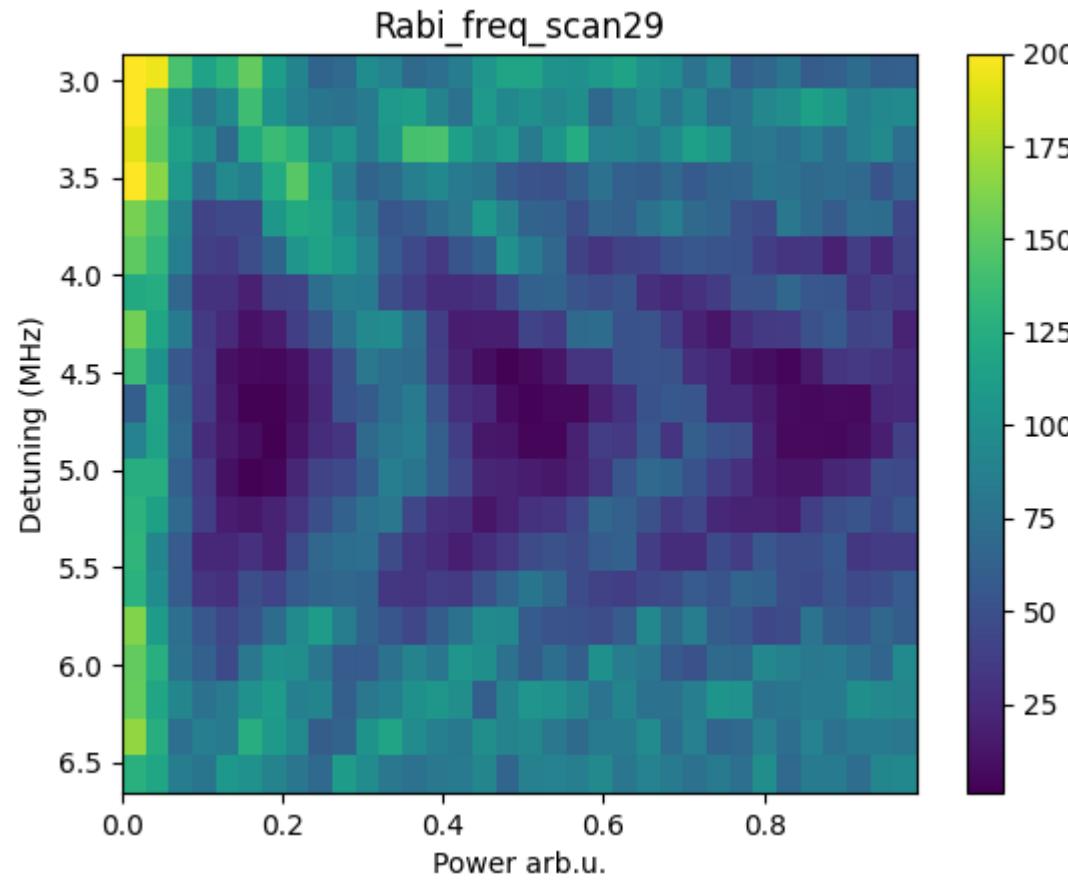
R. Finkelstein et al., Phys. Rev X 11, 011008 (2021)

Rydberg Rabi oscillations

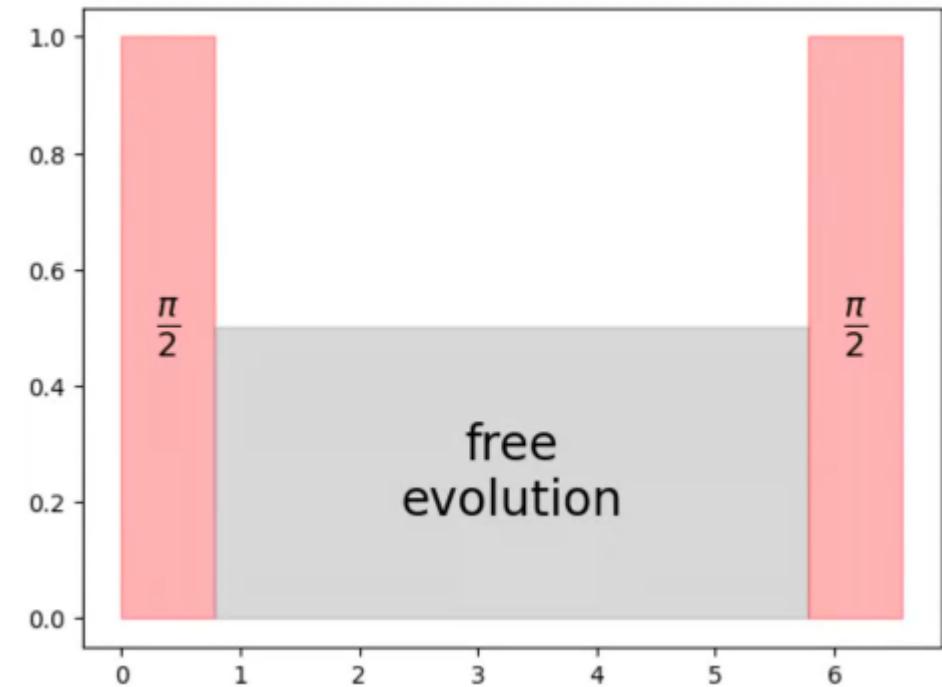
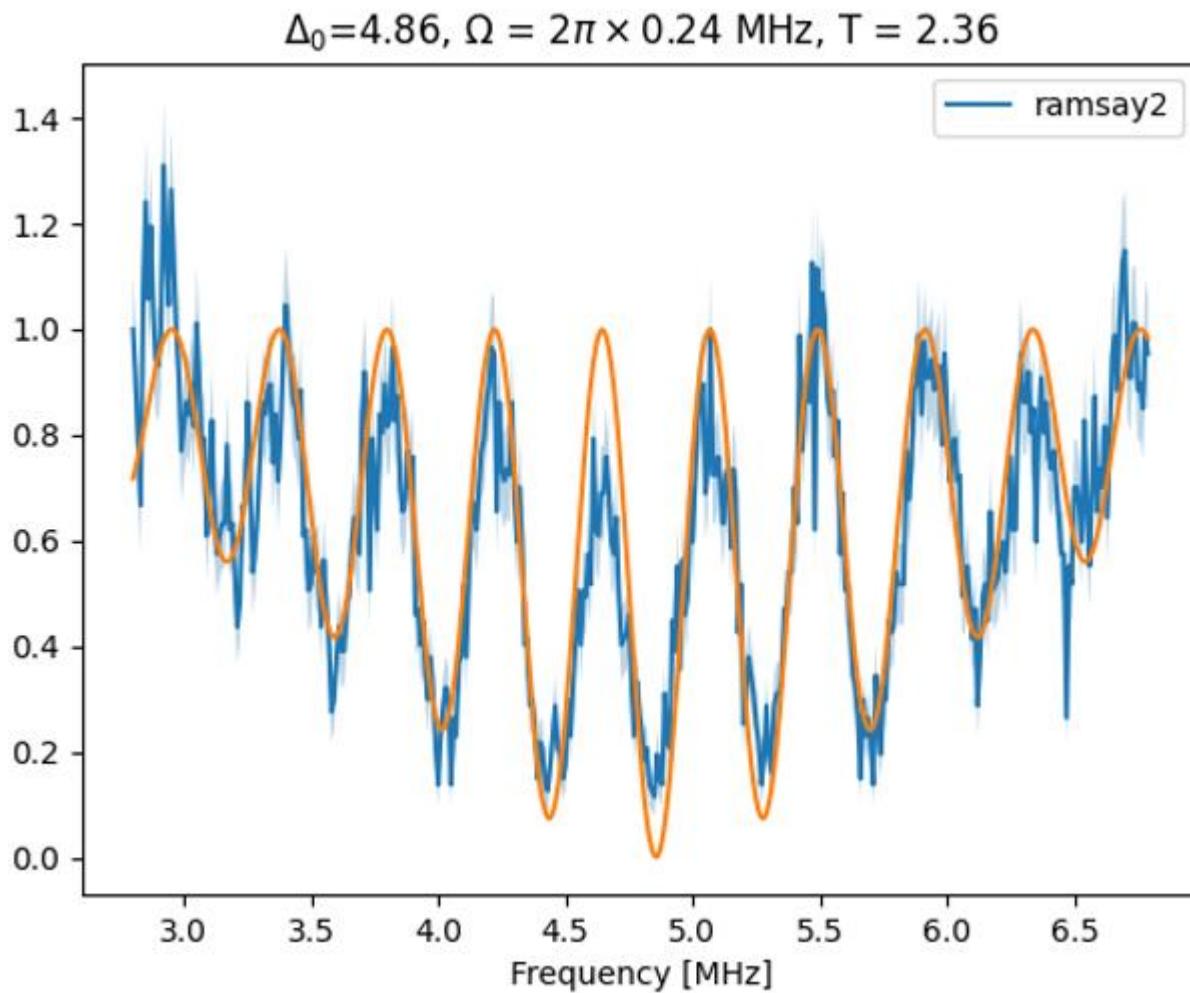


work in progress...

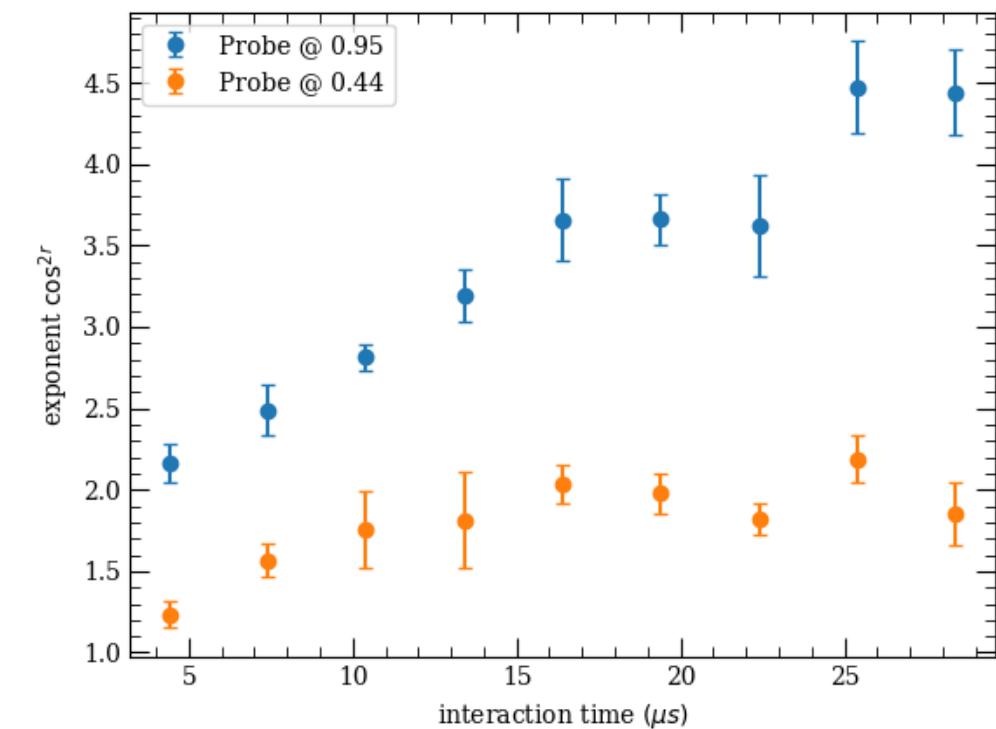
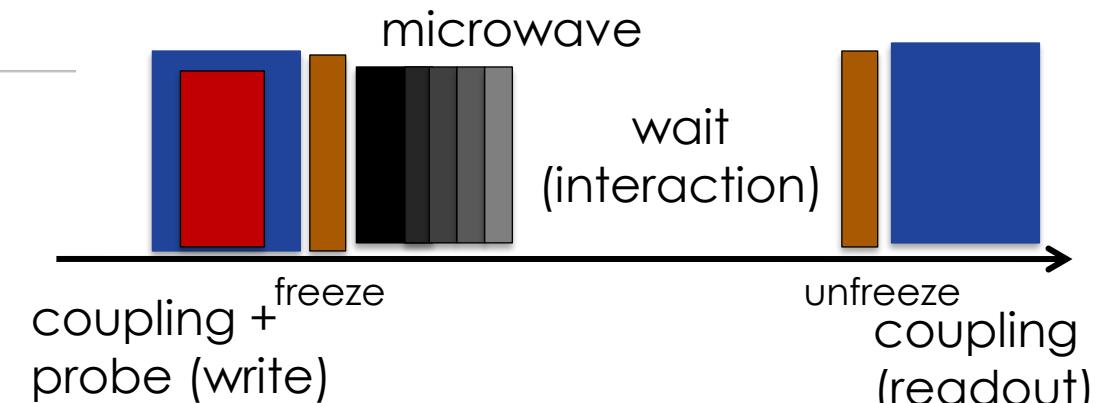
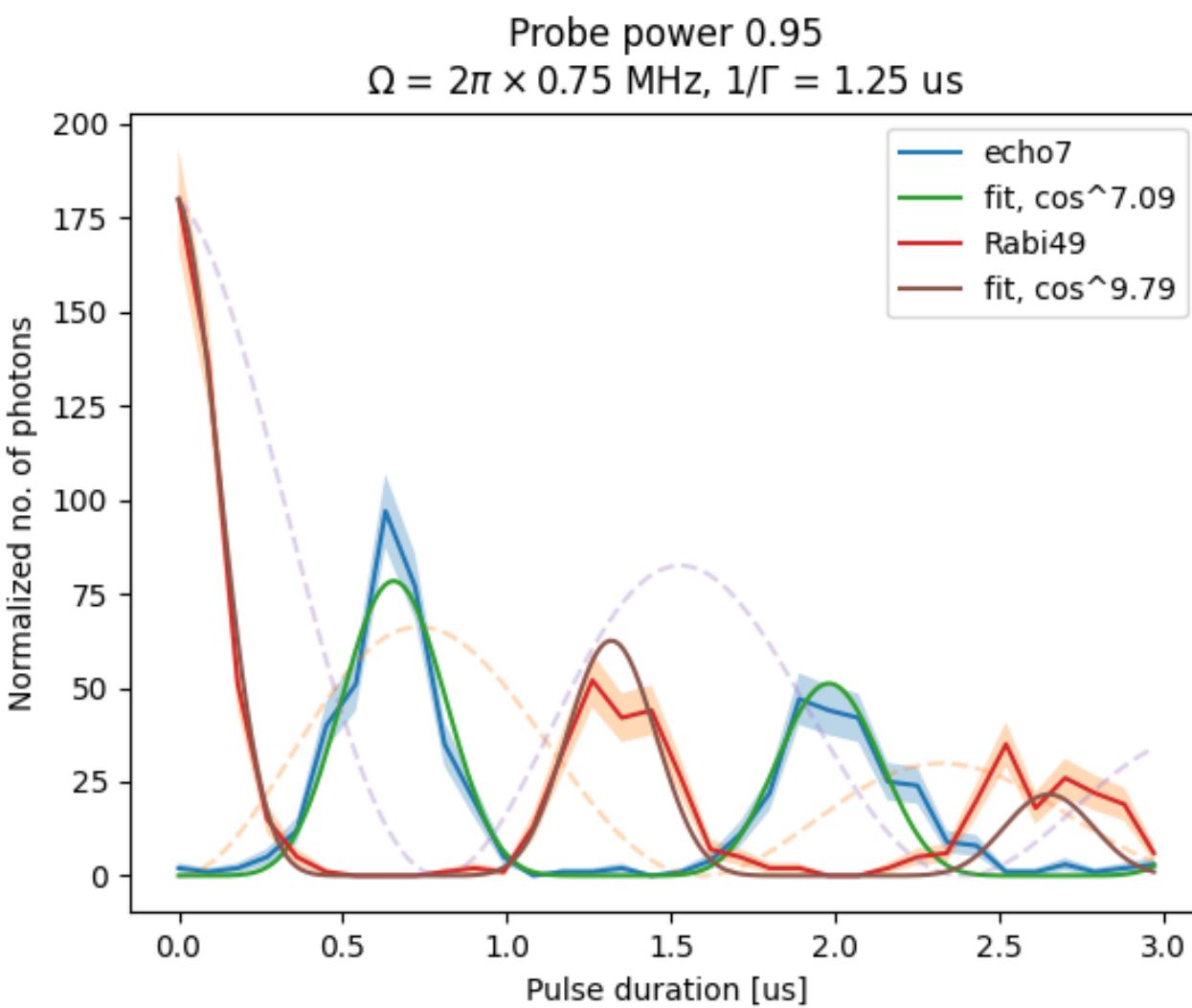
Rydberg Rabi oscillations – qubit analysis picture



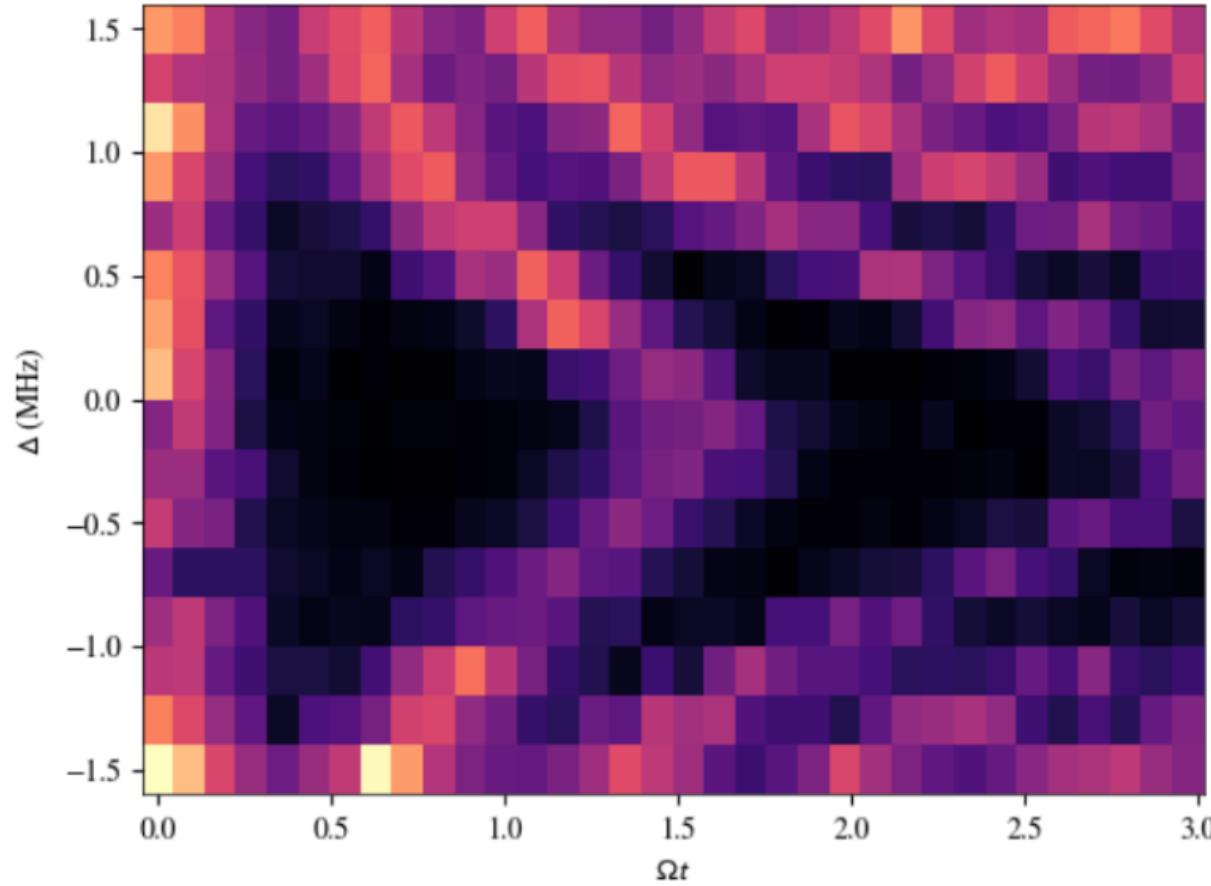
Ramsey fringes



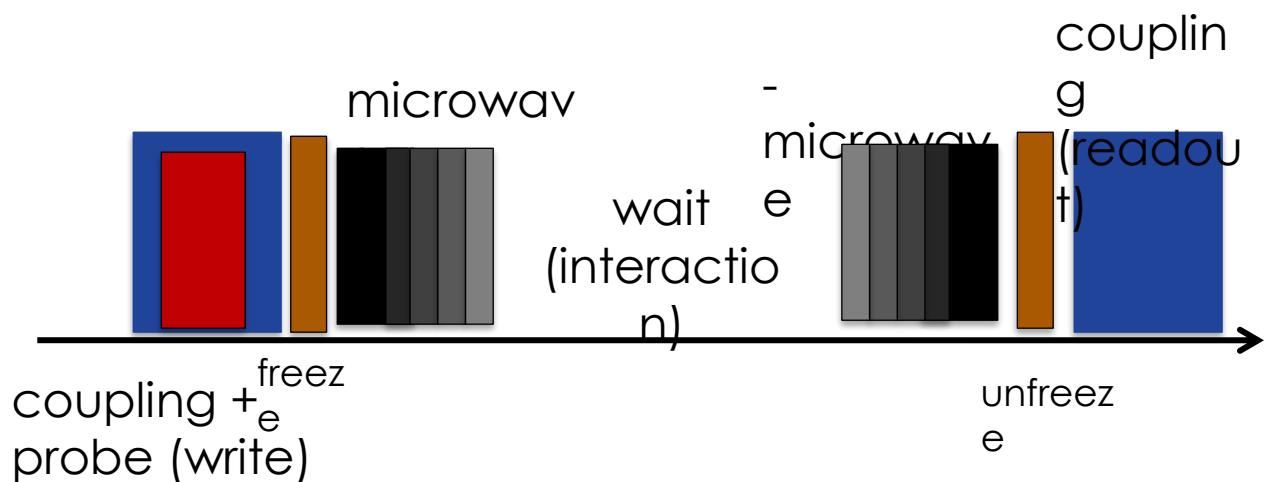
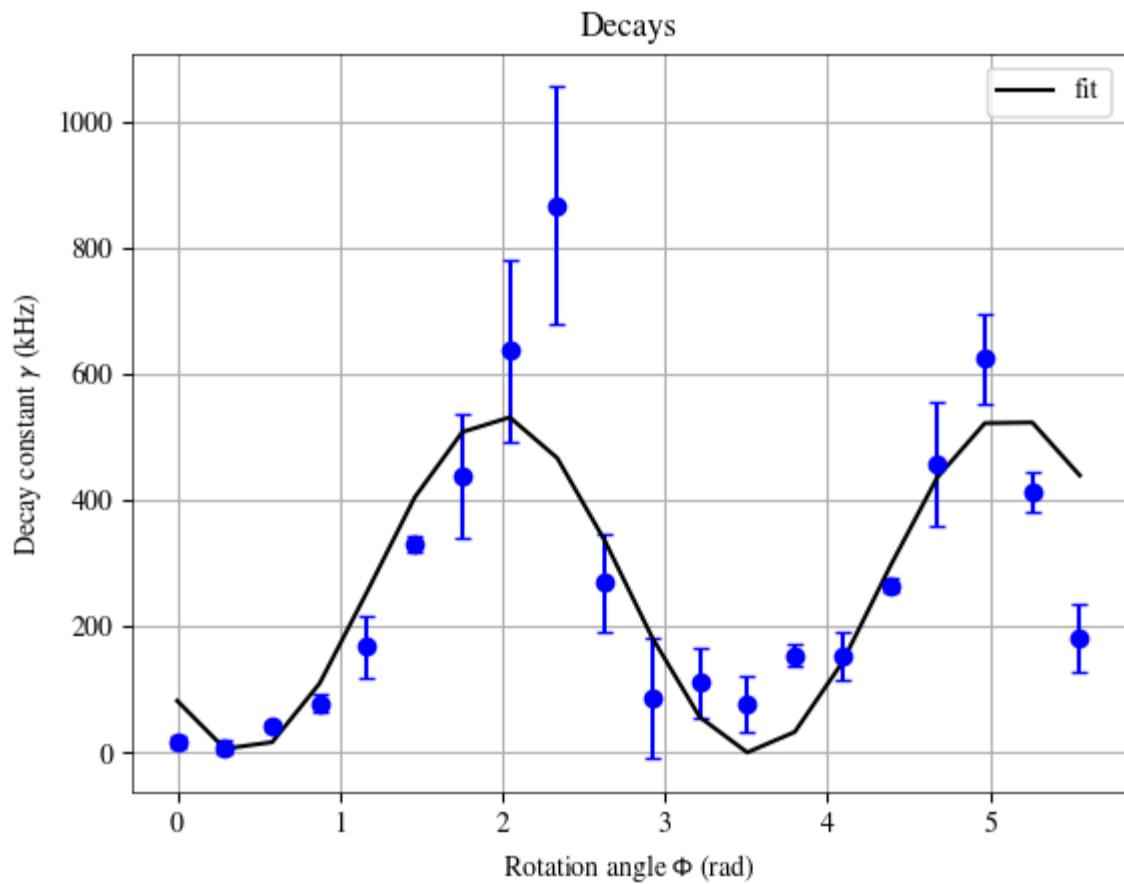
E-field metrology



Ramsey super-fringes



Many-body decoherence

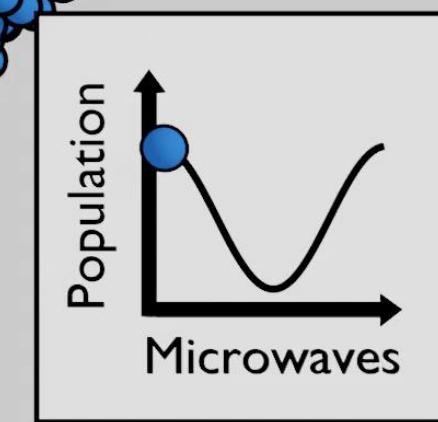
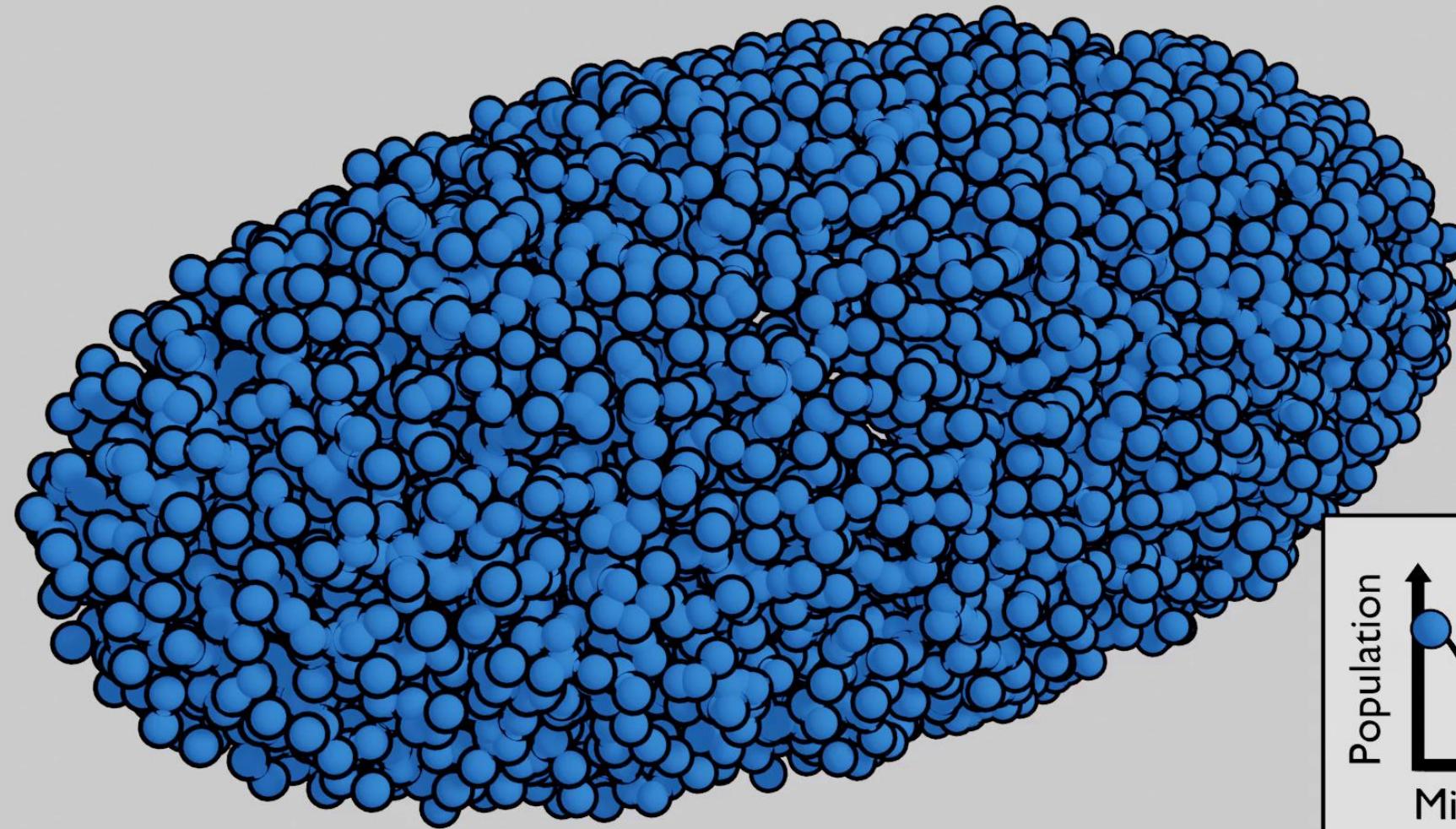


$$\hat{H} = \sum_q V(q) \sum_{\kappa, k} \hat{d}_\kappa \hat{p}_k \hat{d}_{\kappa+q}^\dagger \hat{p}_{k-q}^\dagger \rightarrow \hat{L} = \hat{p} \hat{d}$$

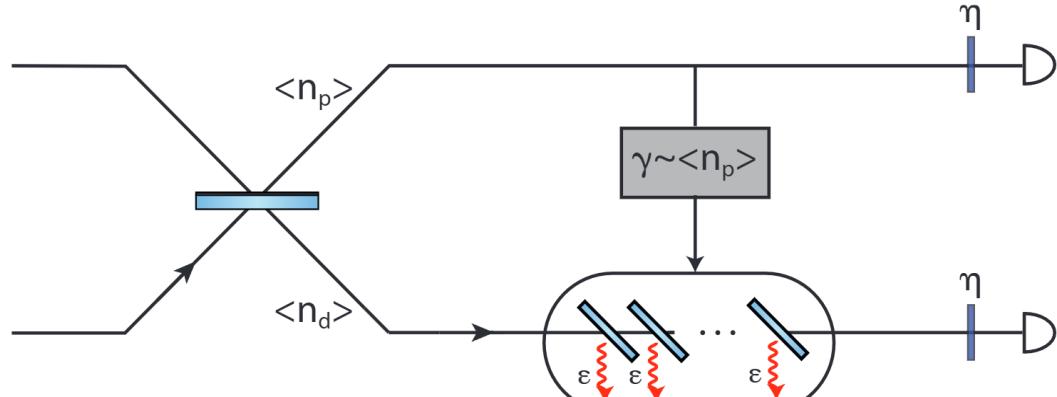
The interaction ($V(r) \sim 1/r^3$) scrambles the phase of the spin-waves \rightarrow scattering into different \mathbf{k} -modes, but only modes with specific \mathbf{k} ($\mathbf{k}=0$) are coupled to light!
In result we get losses (modes with $\mathbf{k}=0$):

$$\hat{d}' \rightarrow \hat{d} \exp\left(-\frac{\gamma\tau}{2}\hat{p}^\dagger \hat{p}\right) \quad \hat{p}' \rightarrow \hat{p} \exp\left(-\frac{\gamma\tau}{2}\hat{d}^\dagger \hat{d}\right)$$

„Microscopic” picture



Quantum metrology context

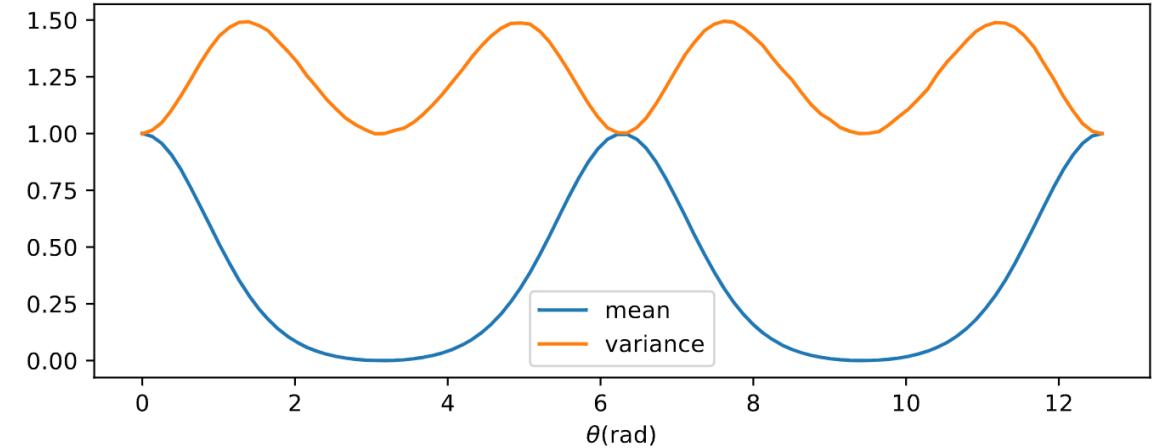


$$\begin{cases} \hat{d}' \rightarrow \hat{d} \exp\left(-\frac{\gamma\tau}{2}\hat{p}^\dagger\hat{p}\right) + \hat{v}_1\sqrt{1 - \exp(-\gamma\tau\hat{p}^\dagger\hat{p})} \\ \hat{p}' \rightarrow \hat{p} \exp\left(-\frac{\gamma\tau}{2}\hat{d}^\dagger\hat{d}\right) + \hat{v}_2\sqrt{1 - \exp(-\gamma\tau\hat{d}^\dagger\hat{d})} \end{cases}$$

$$\begin{cases} \hat{d}'' \rightarrow \sqrt{\eta}\hat{d}' + \sqrt{1-\eta}\hat{c}_1, \\ \hat{p}'' \rightarrow \sqrt{\eta}\hat{p}' + \sqrt{1-\eta}\hat{c}_2 \end{cases}$$

$$p(n) = \sum_{k=0}^{\infty} \frac{|p|^{2k} \left(|\delta|^2 e^{-2\gamma k}\right)^n e^{-|\delta|^2 e^{-2\gamma k} - |p|^2}}{k! n!}$$

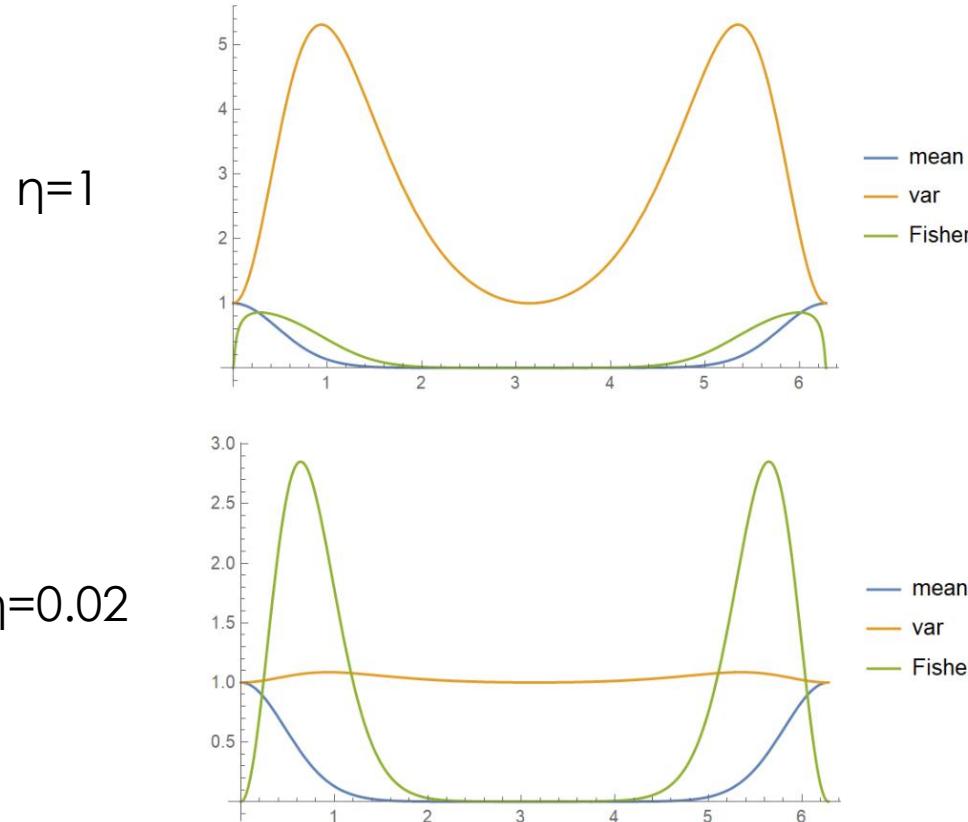
work in progress...



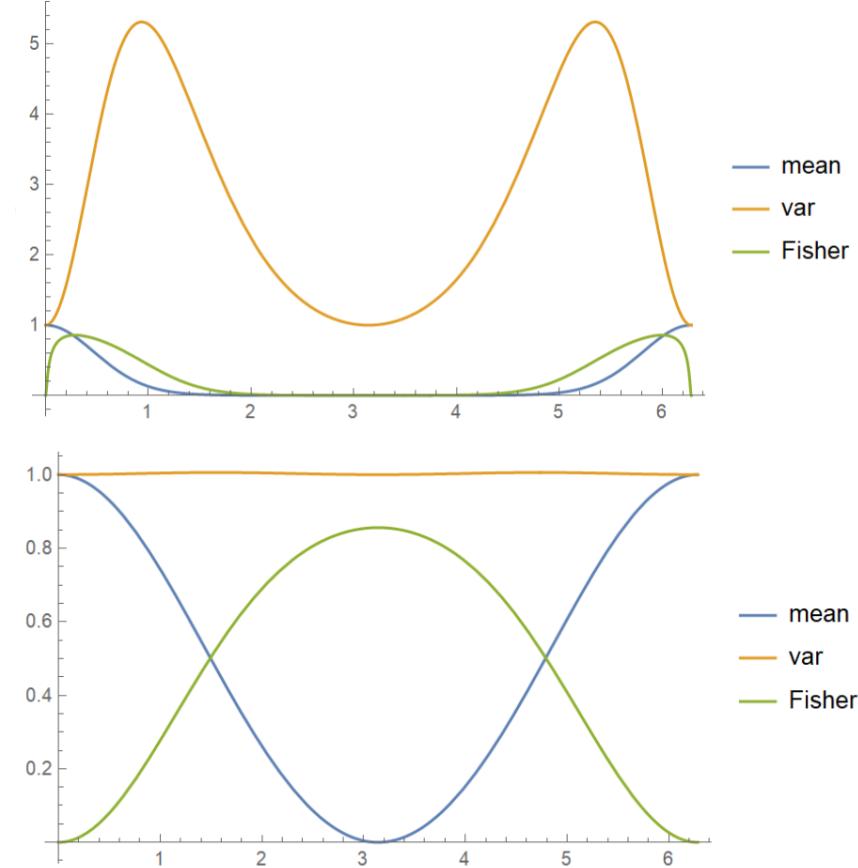
For $\eta=1$ we get sharper peaks but we pay for that with increased variance!
So where is the gain?

Losses at the input vs losses at the output

Losses at the output (after interaction)



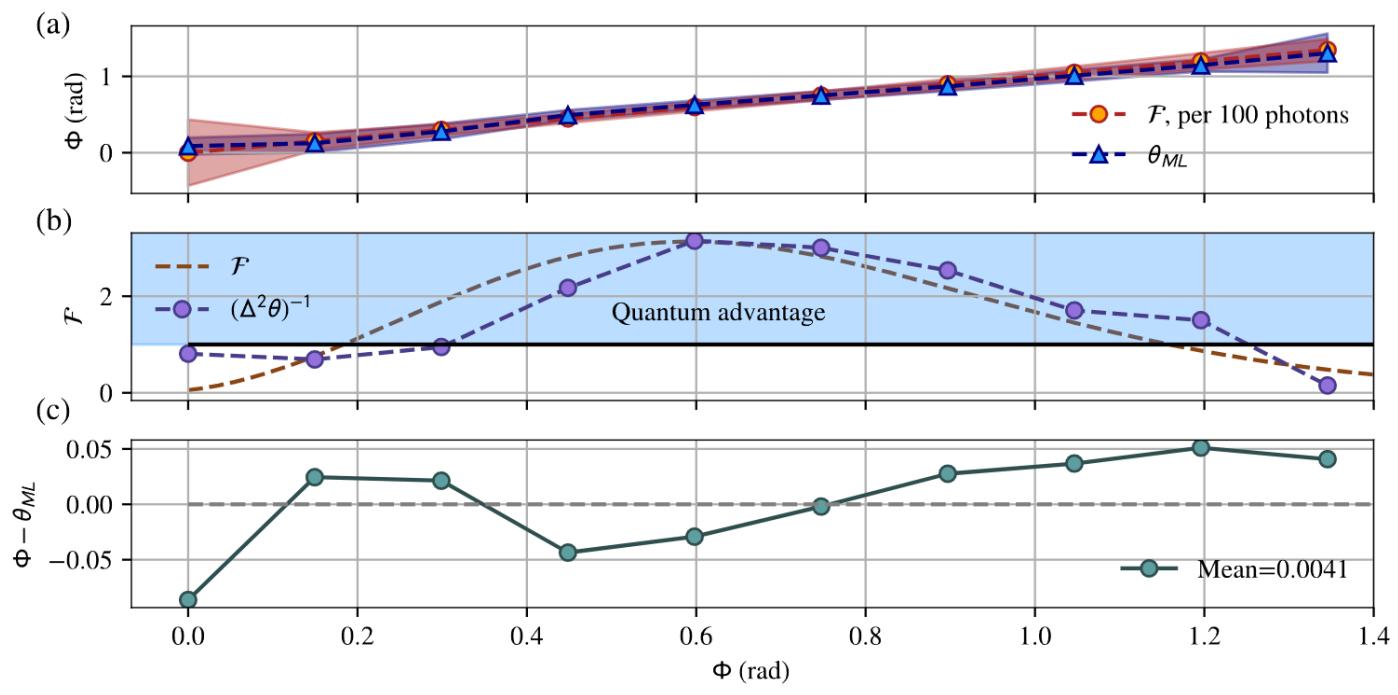
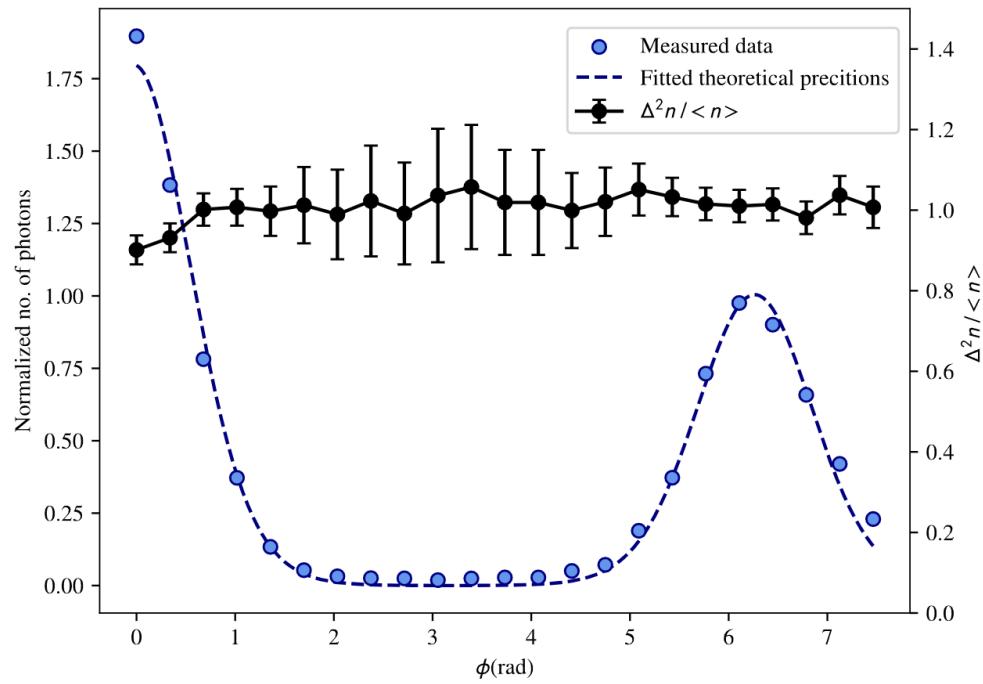
Losses at the input (before interaction)



Fisher and mean divided by η
work in progress...

$$d\mathcal{F} = \left(\frac{\partial p_{n_d}(k, \theta)}{\partial \theta} \right)^2 \frac{1}{p_{n_d}(k, \theta)} \quad \mathcal{F} = \sum_{k=0}^{\infty} d\mathcal{F}$$

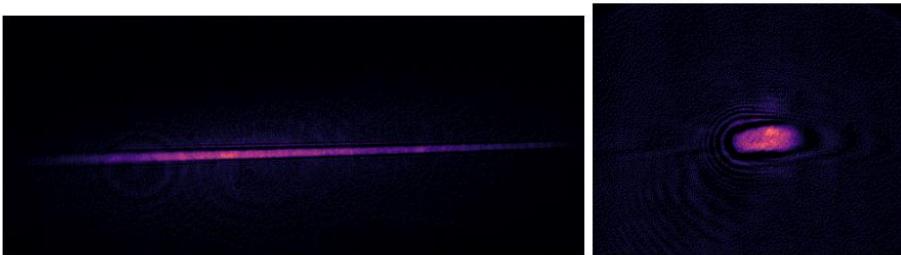
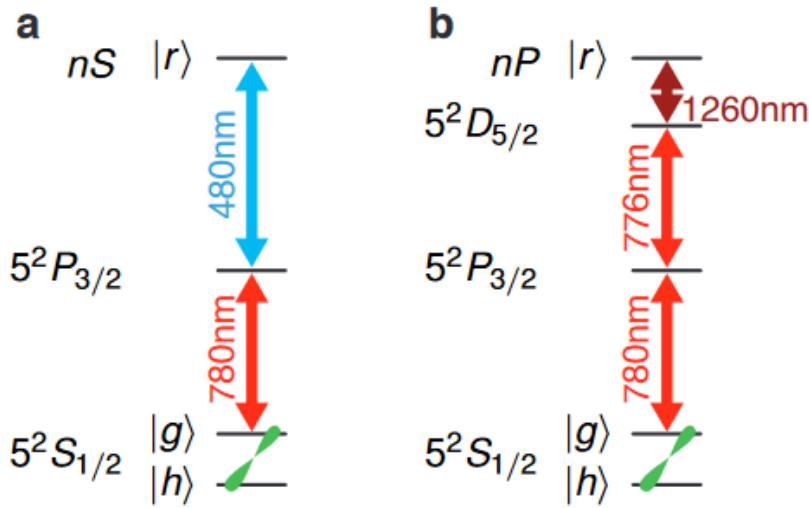
With losses there is an advantage!



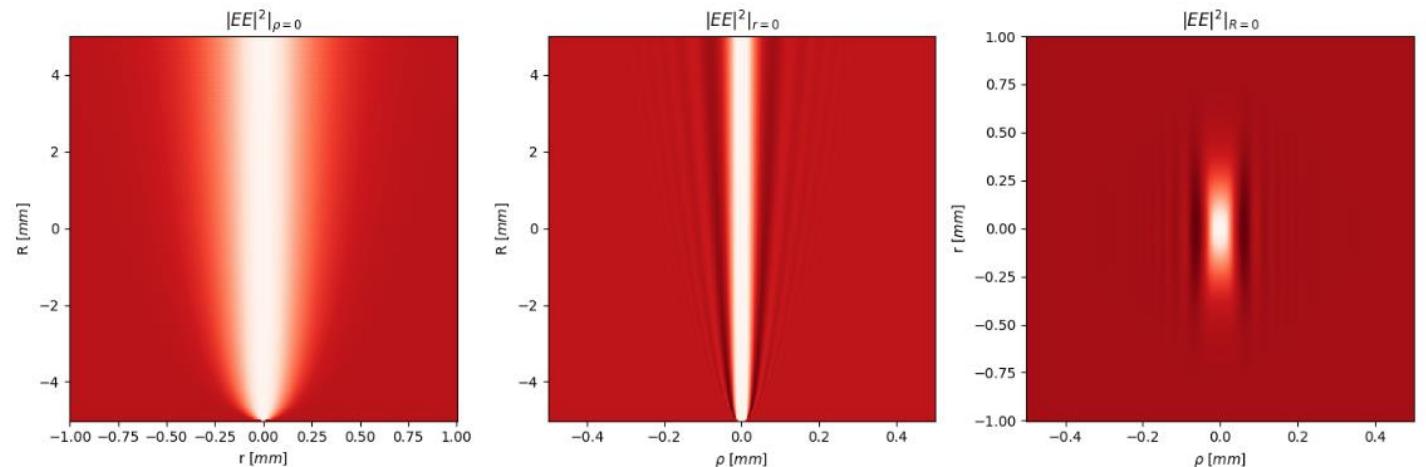
work in progress...

Outlook: 3d Rydberg polaritons

Blockade regime

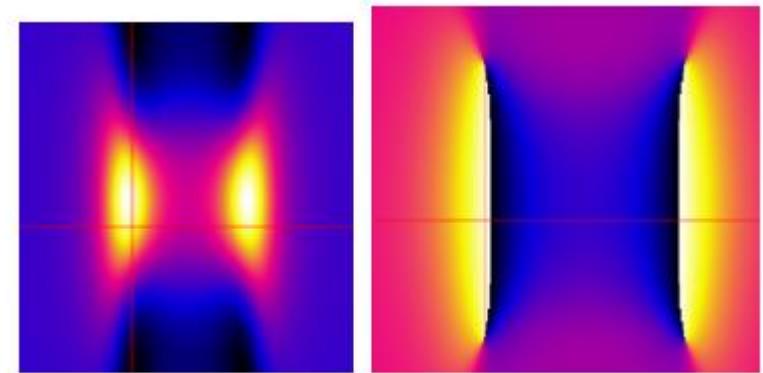
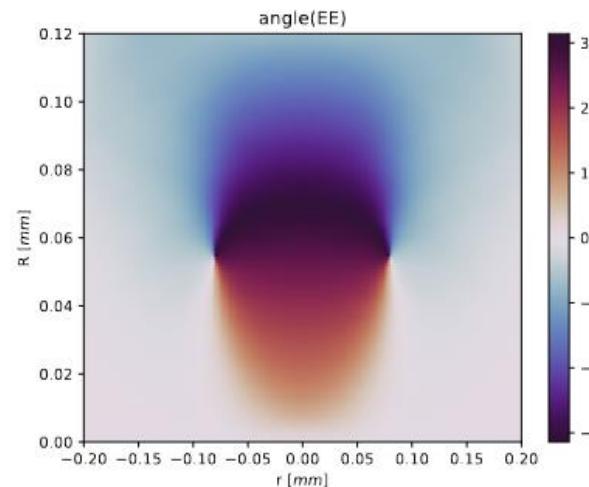
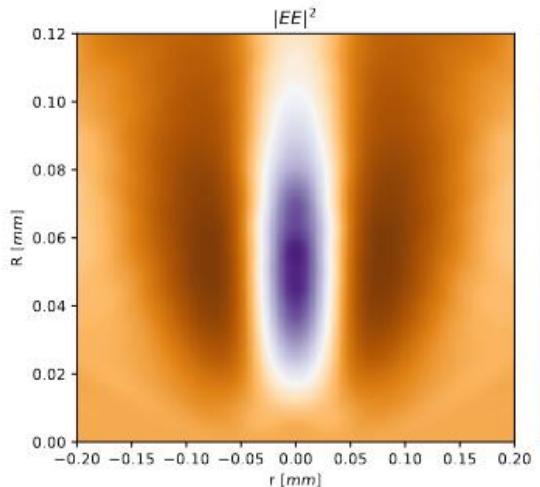


$$\begin{aligned} \frac{\partial EE}{\partial R} = C_D & \left(\frac{\partial^2 EE}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial EE}{\partial \rho} \right) - \left(C_{VR} V(r, \rho) - C_{Vr} \frac{\partial^2 V(r, \rho)}{\partial r^2} \right) EE + \\ & + C_r \frac{\partial^2 EE}{\partial r^2} + C_{Vr} \left(\frac{\partial^2 EEV(r, \rho)}{\partial r^2} - 2 \frac{\partial (EE \frac{\partial V(r, \rho)}{\partial r})}{\partial r} \right). \end{aligned}$$



Outlook: 3d Rydberg polaritons

Attractive potential regime



1d version of vortices – see below

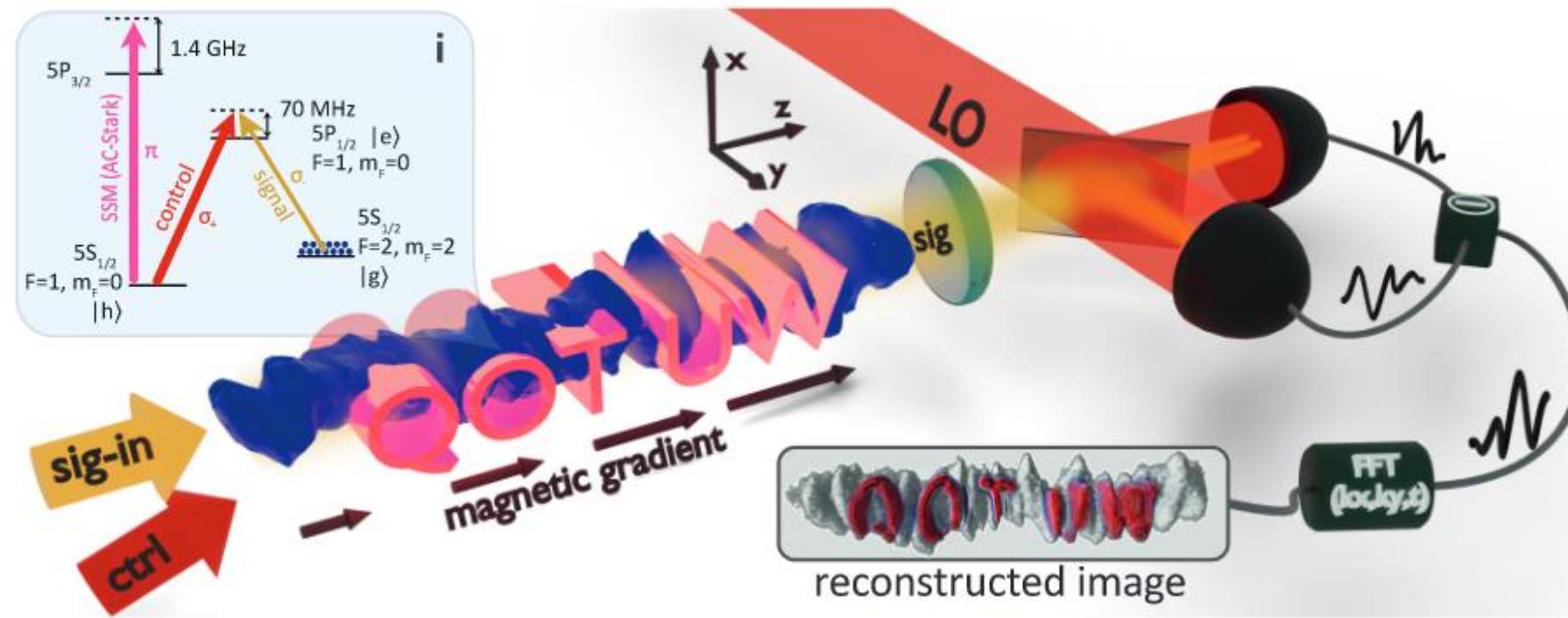
3d version of vortices

Quantum vortices of strongly interacting photons

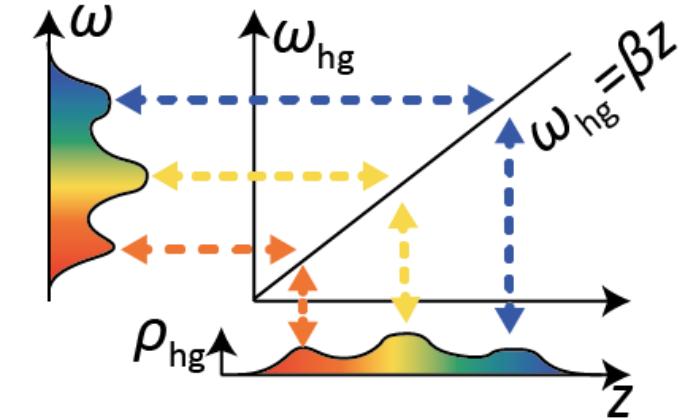
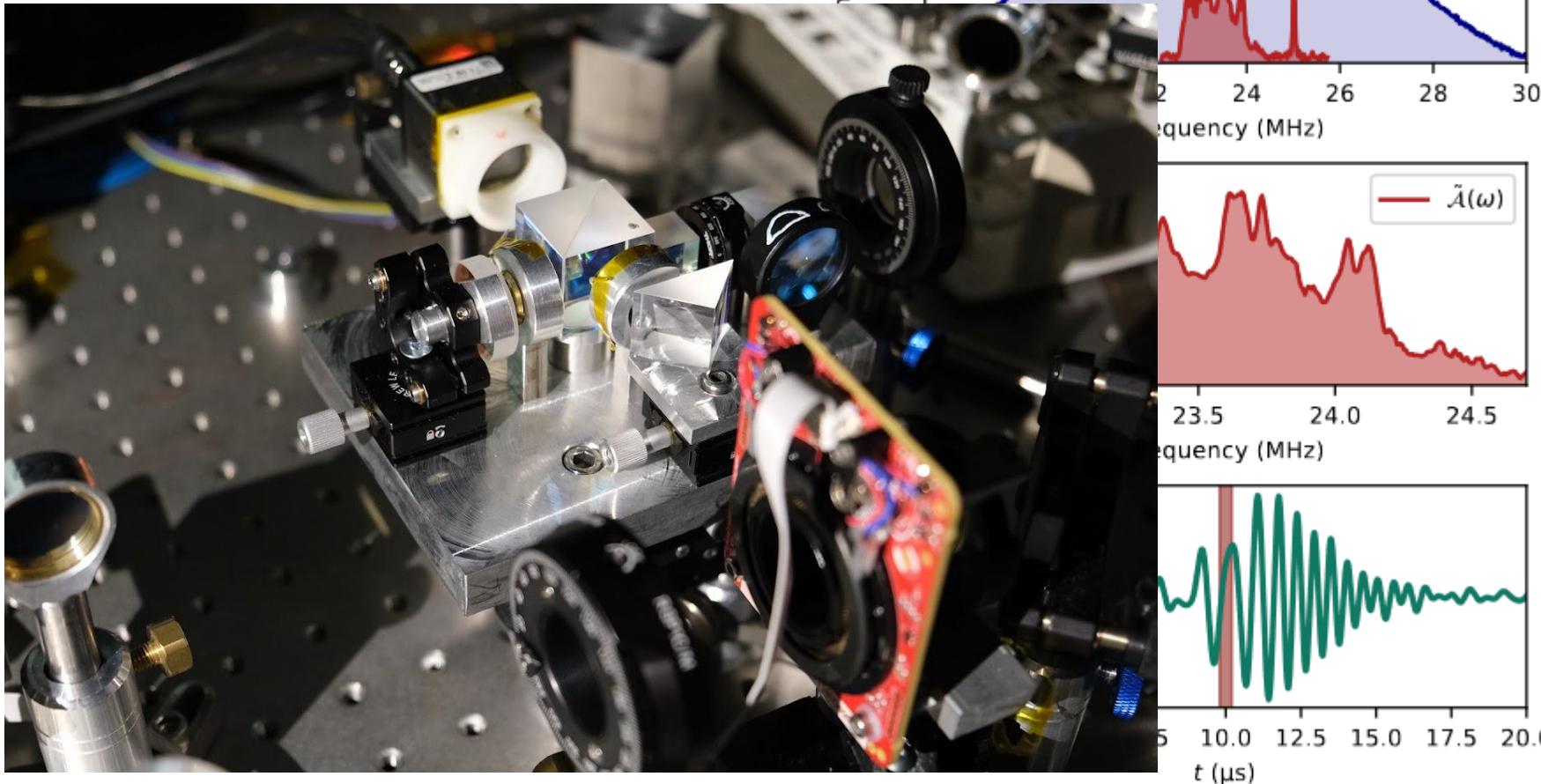
LEE DRORI , BANKIM CHANDRA DAS , TOMER DANINO ZOHAR , GAL WINER , EILON POEM , ALEXANDER PODDUBNY , AND OFER FIRSTENBERG

[Authors Info & Affiliations](#)

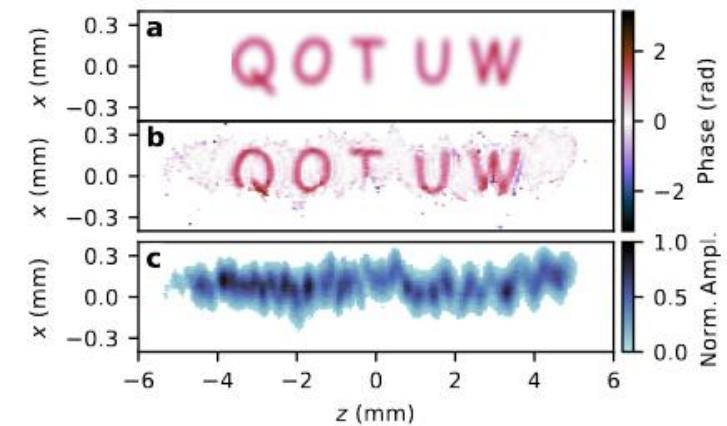
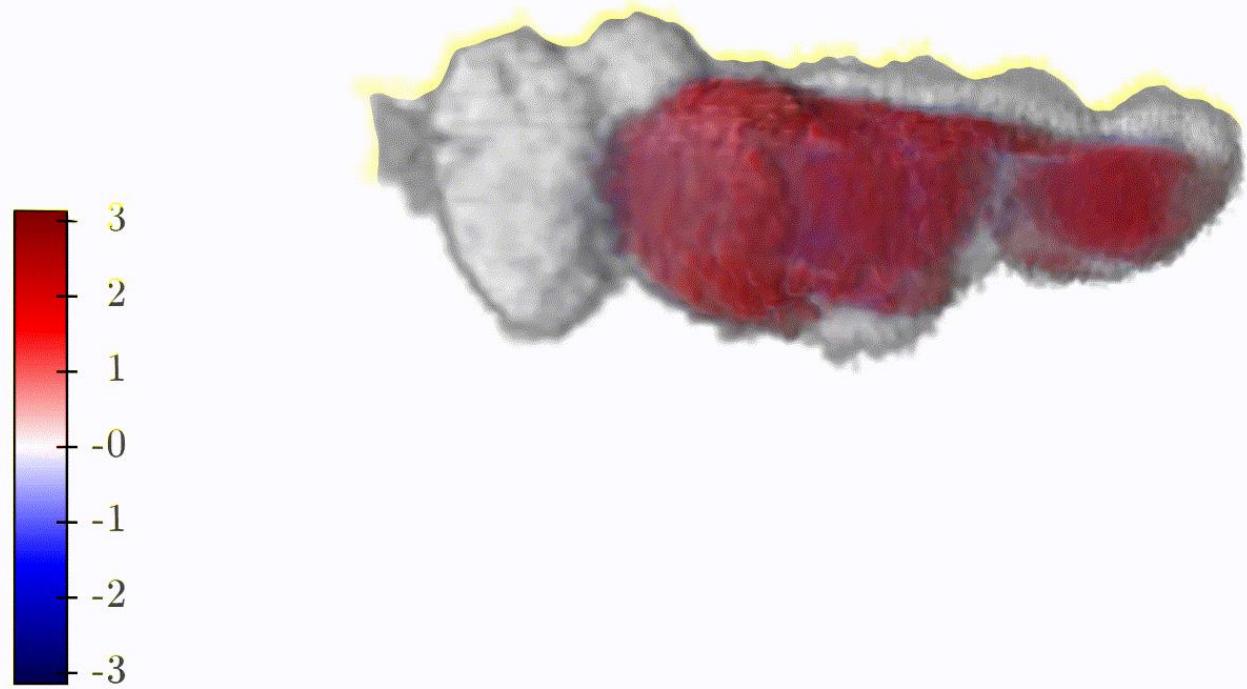
Tomography



Tomography (1D \rightarrow 3D)



Tomography (3D)



Rydberg blockade tomography

