QUANTUM ELECTROMAGNETICS
16-20 NOVEMBER

European School of Antennas and Propagation
The EurAAP School

uc3m Universidad Carlos III de Madrid

DIRAQ
DESIGN AND RESEARCH IN RADIOFREQUENCY AND QUANTUM APPLICATIONS

LECTURE 1 · 2 HOURS

Rydberg atoms:
giant atoms for quantum technologies

Why highly excited atoms are the most microwave-friendly quantum systems we know

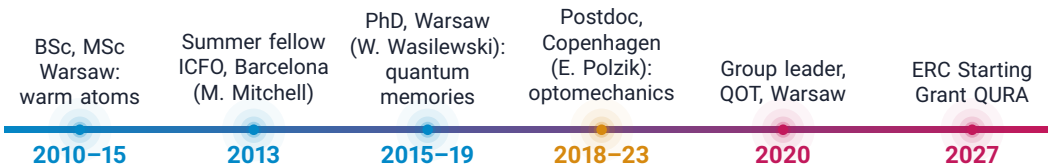
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University of Warsaw · Faculty of Physics, University of Warsaw · CeNT Centre of New Technologies · QOT Centre for Quantum Optical Technologies · QOD

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European Funds for Smart Economy · Republic of Poland · Co-funded by the European Union

About me



Michał Parniak

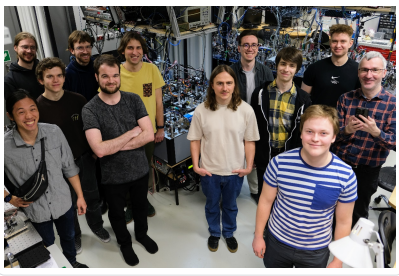
Assistant Professor, Faculty of Physics, and Group Leader,
Centre for Quantum Optical Technologies, University of Warsaw

Atoms, then membranes, now **atoms as antennas**: one thread, how light talks to matter.

Along the way

- Habilitation 2023; professor at UW since 2024
- Frank Wilczek Prize, 2022
- W. Rubinowicz Prize, Polish Physical Society, 2023
- Lindau Nobel Laureate Meeting, 2024

My lab: Quantum Optical Devices Lab



Centre for Quantum Optical Technologies, CeNT,
University of Warsaw

Funding: ERC, NCN, FNP (FENG), ESA

What we do

- **Rydberg sensors** of microwave, mm-wave and THz fields
- Microwave-to-optical **conversion** and photon counting
- **Cold-atom** quantum memories and metrology
- From table to instrument: ESA RYDAR, and the spin-off **QuRA.space**

Team: M. Mazelanik, W. Wasilewski; PhD students S. Borówka, M. Jastrzębski, B. Kasza, G. Ko, W. Krokosz, S. Kurzyna, B. Niewelt, J. Nowosielski **www.qodl.eu**

Plan of the lecture

01 What makes Rydberg atoms special

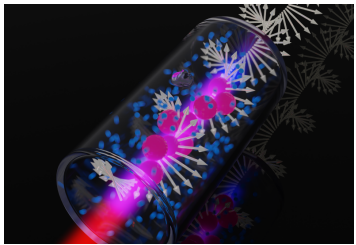
scaling laws (n^2 dipoles, n^7 polarisability, n^3 lifetimes), a dense ladder of transitions from MHz to THz; the atom as a resonant dipole antenna

02 Controlling Rydberg atoms with light

laser excitation ladders, EIT and Autler–Townes; hot vapours, cold clouds, tweezer arrays; collective excitations and quantum memories

03 Interactions and quantum technologies

dipole–dipole interactions and the Rydberg blockade; gates for neutral-atom quantum computers, single-photon nonlinearities, cavity QED with microwave photons



Why should an antenna engineer care about atoms?

A metal antenna

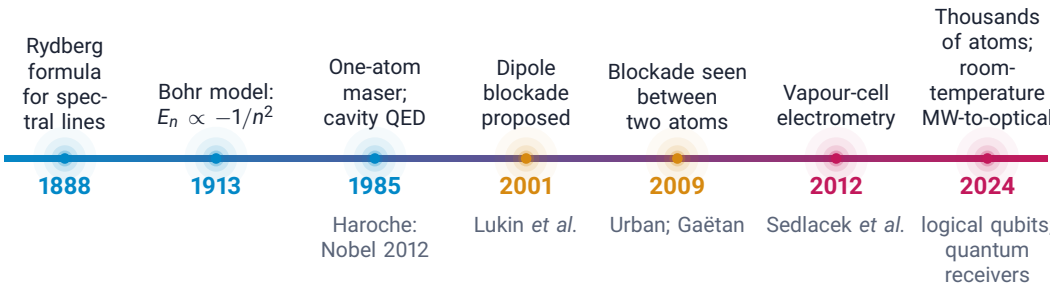
- Free electrons driven by the field; size tied to λ
- Calibrated against another antenna
- Metal disturbs the field it measures
- Needs an RF front-end: LNA, mixer, ADC
- Noise floor: $k_B T$ of the chain

A cloud of Rydberg atoms

- Bound electrons on a resonant transition; size independent of λ
- Calibrated by atomic constants and h
- Glass and light only: nearly transparent to RF
- Readout is optical: the RF chain moves to the laser side
- Noise floor: quantum noise of light and atoms

Theme of both lectures: the same Maxwell equations, but the receiving element is a **quantum two-level system** instead of a conductor.

Rydberg physics in one timeline



Rydberg atoms have repeatedly served as the **cleanest available testbed** for light–matter interaction. Now they are also a **technology**.

Units we will use

Atomic scales

- Bohr radius $a_0 = 0.052\,9$ nm
- Dipoles in $ea_0 = 8.48 \times 10^{-30}$ C m (2.54 D)
- Rydberg energy $\mathrm{Ry} = 13.6$ eV

Frequencies, not energies

- Rates as $\Omega/2\pi$ in Hz, so that $E = h\nu$
- Rabi frequency: how fast the field drives the transition, $\Omega = dE/\hbar$

The one number worth remembering

$$\frac{ea_0}{h} = 12.8\,\mathrm{kHz/(V/m)}$$

Back of the envelope

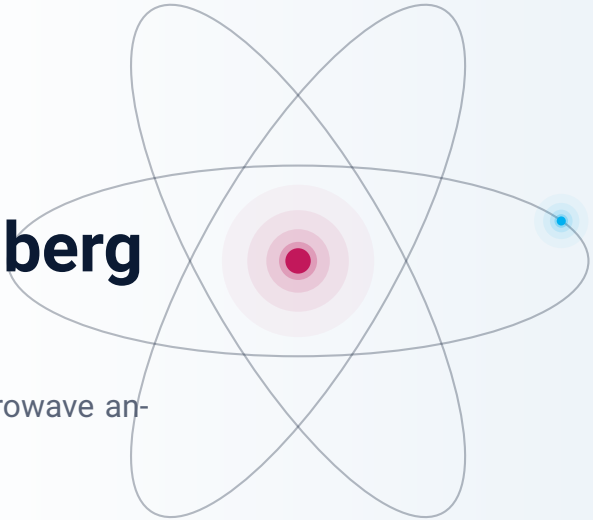
A dipole of 1450 ea_0 (Rb $50S \to 50P$) in 1 mV/cm = 0.1 V/m:
$\Omega/2\pi = 1450 \times 12.8\,\mathrm{kHz} \times 0.1 = 1.85\,\mathrm{MHz}$.

Easily resolved with lasers.

01

What makes Rydberg atoms special

Scaling laws and the atom as a microwave antenna



The hydrogen-like picture

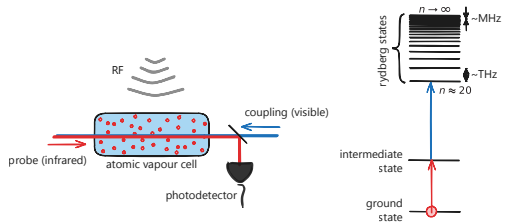
One valence electron far from a singly charged core:

$$E_n = -\frac{\mathrm{Ry}}{n^2}, \qquad r_n \sim n^2 a_0$$

Neighbouring levels:

$$E_{n+1} - E_n \approx \frac{2\,\mathrm{Ry}}{n^3}$$

$n = 50$: binding energy 6 meV, below $k_B T$ at 300 K (25 meV); level spacing in the **microwave** band.



laser-excited atoms in a vapour cell, and the ladder of states

Allinson *et al.*, arXiv:2601.20631 (2026)

The scaling laws, derived in four lines

1. Circular orbit, Coulomb force:

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

2. Bohr quantisation: $mvr = n\hbar$
3. Divide (1) by (2), with $\alpha = e^2/4\pi\epsilon_0\hbar c$:

$$v = \frac{\alpha c}{n}, \qquad r = n^2 a_0$$

4. Total energy is half the potential energy:

$$E_n = -\tfrac{1}{2}mv^2 = -\frac{\mathrm{Ry}}{n^2}$$

Bonus: the correspondence principle

Level spacing:

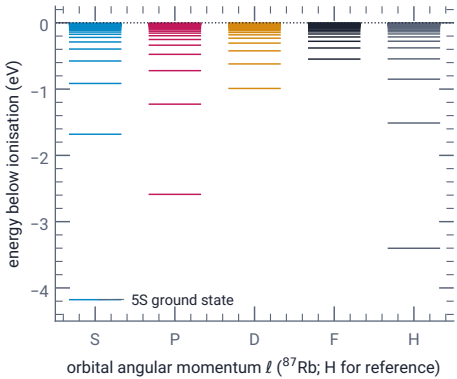
$$\frac{E_{n+1} - E_n}{\hbar} \approx \frac{2\mathrm{Ry}}{\hbar n^3} = \frac{\alpha c}{a_0 n^3}$$

Orbital frequency:

$$\frac{v}{r} = \frac{\alpha c/n}{n^2 a_0} = \frac{\alpha c}{a_0 n^3}$$

They are **equal**. The microwave transition frequency is how often the electron goes round: a Rydberg atom is a classical rotating dipole, of size $d \sim er \propto n^2$.

Real atoms: quantum defects



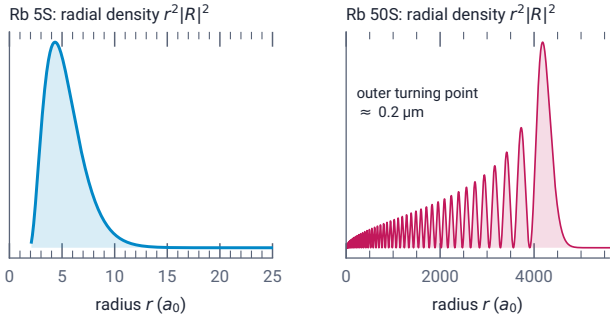
Low-ℓ orbits penetrate the ion core and feel more charge:

$$E_{n\ell j} = -\frac{\mathrm{Ry}^*}{(n-\delta_{\ell j})^2}$$

Rb	S	$P_{3/2}$	$D_{5/2}$	F
$\delta_{\ell j}$	3.13	2.64	1.35	0.02

- Degeneracy lifted: a **rich set of frequencies** instead of one hydrogen ladder
- $\ell \gtrsim 4$ are almost hydrogenic

How big is a Rydberg atom?



- Ground state: a few a_0. Rydberg: thousands of a_0
- $n = 100$: about 1 μm, the size of a bacterium

- The electron spends its time at the outer turning point
- Hence the huge dipole moment

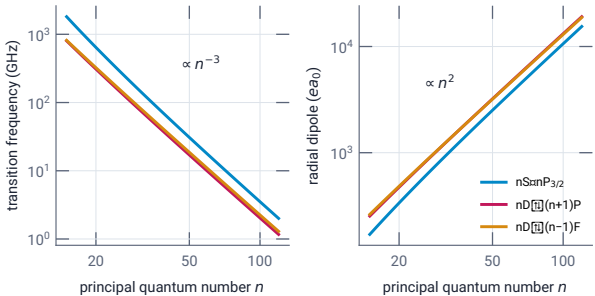
Scaling laws

Property	**Scaling**	$n = 30$	$n = 50$	$n = 70$	$n = 100$
Binding energy (meV)	n^{-2}	18.8	6.2	3.0	1.5
Orbit size $2n^2 a_0$ (μm)	n^2	0.1	0.26	0.5	1.1
$nS \to nP_{3/2}$ frequency (GHz)	n^{-3}	162	30.8	10.7	3.5
Radial dipole (ea_0)	n^2	840	2500	5100	10 600
Lifetime at 0 K (μs)	n^3	24	129	374	1 140
Lifetime at 300 K (μs)	$\sim n^2$	16	63	147	347
Ionisation field (V/cm)	n^{-4}	400	51	13	3
Polarisability	n^7				
$\lvert C_6 \rvert$ (GHz μm^6)	n^{11}	0.02	15	860	56 000

One knob, n, moves all of these together: every design choice is a trade-off.

Rb $nS_{1/2}$, computed with ARC. Ionisation field: classical $1/16n^4$ in atomic units, with n not n^*

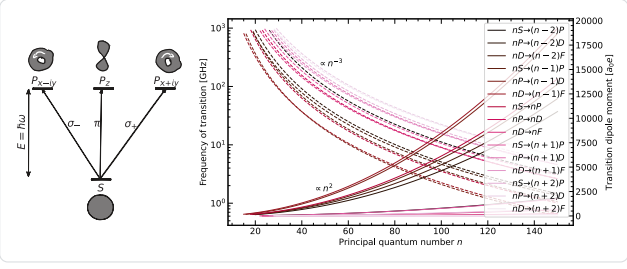
A dense ladder of transitions: MHz to THz



- Lines cover L-band to THz: **choose the band by choosing n and ℓ**
- Between lines: DC Stark tuning, or off-resonant detection

- High frequency means small dipole: sensitivity falls as frequency rises
- $f \propto n^{-3}$, $d \propto n^2$, so $d \propto f^{-2/3}$

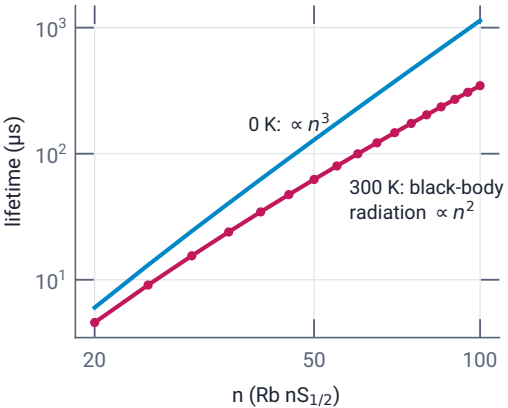
Selection rules: the atom sees polarisation



Allinson *et al.*, arXiv:2601.20631 (2026)

- Electric dipole transitions: $\Delta\ell = \pm 1$
- Three elementary dipoles: π (linear along the quantisation axis), $\sigma^{\pm}$ (circular)
- The atom is a **polarisation-selective** element, with the axis set by a small bias field or by the laser polarisation
- Far from resonance the response falls as $1/\Delta$, like a driven resonator

Lifetimes, and why temperature matters



- Radiative decay to low states: $\tau_0 \propto n^3$
- At room temperature **black-body photons** drive Rydberg–Rydberg transitions
- Microwave modes are highly occupied:

$$\bar{n} = \frac{1}{e^{hf/k_BT} - 1} \approx \frac{k_BT}{hf}$$

Back of the envelope

At 30.8 GHz and 300 K: $\bar{n} \approx 200$ photons per mode.

Thermal radiation is not a small effect: it halves the 50S lifetime, from 129 to 63 μs.

Lifetimes, derived

Radiative decay: $\tau_0 \propto n^3$

1. Einstein rate: $A \propto \omega^3 |d|^2$
2. Strongest decay is to *low* states: ω does not depend on n
3. The dipole needs overlap with the small low state, near the core, where

$$|\psi_n(\text{core})|^2 \propto n^{-3}$$

$$A \propto n^{-3} \;\Rightarrow\; \tau_0 \propto n^3$$

Black-body decay: $\tau_{\text{bb}} \propto n^2$

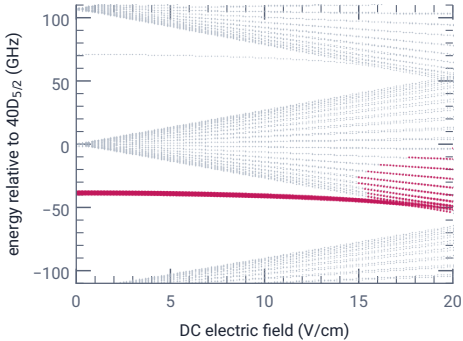
1. Stimulated rate to a neighbour: $\Gamma \sim A\,\bar{n}$
2. Neighbours: $A \propto \omega^3 d^2 \propto n^{-9} n^4 = n^{-5}$
3. Thermal photons: $\bar{n} \approx k_B T/\hbar\omega \propto n^3$

$$\Gamma_{\text{bb}} = \frac{4\alpha^3 k_B T}{3\hbar n^2}$$

Back of the envelope

$n = 50$, 300 K: $8.1 \times 10^3\ \text{s}^{-1}$. ARC: $8.2 \times 10^3\ \text{s}^{-1}$.

Polarisability: extreme DC sensitivity



Rb, $m_j = 5/2$, computed with ARC; red: $40D_{5/2}$ character

- Quadratic shift at low field: $\Delta E = -\frac{1}{2}\alpha F^2$, with $\alpha \propto n^7$
- Linear manifolds of high-ℓ states fan out and cross the low-ℓ states
- Avoided crossings, then field ionisation

Wall charges shift the lines: the *cell* is part of the antenna.

Polarisability and ionisation, derived

Polarisability $\propto n^7$

1. Second-order Stark shift, mainly from the nearest level:

$$\Delta E = -\frac{d^2F^2}{\delta E} \equiv -\tfrac{1}{2}\alpha F^2$$

2. Insert $d \propto n^2$, $\delta E \propto n^{-3}$:

$$\alpha \sim \frac{2d^2}{\delta E} \propto \frac{n^4}{n^{-3}} = n^7$$

Ionisation field $\propto n^{-4}$

1. Coulomb plus uniform field, along z:

$$V(z) = -\frac{e^2}{4\pi\epsilon_0 z} - eFz$$

2. Saddle from $V'(z_s) = 0$: $z_s = \sqrt{e/4\pi\epsilon_0 F}$

$$V(z_s) = -2\sqrt{e^3F/4\pi\epsilon_0}$$

3. Set equal to $E_n = -\mathrm{Ry}/n^2$:

$$F_{\mathrm{ion}} = \frac{1}{16n^4}\ \text{a.u.} = \frac{5.14\times10^9\ \text{V/cm}}{16n^4}$$

$n = 50$: 51 V/cm, easily applied.

Hands-on: ARC gives you the numbers


```python
from arc import Rubidium87

rb = Rubidium87()
n = 50                                  # try 30, 70, 100

# n, l, j -> n', l', j'  (Hz)
f = rb.getTransitionFrequency(n, 0, 0.5, n, 1, 1.5)
# n, l, j, mj -> n', l', j', mj', q  (e a0)
d = rb.getDipoleMatrixElement(n, 0, 0.5, 0.5, n, 1, 1.5, 1.5, 1)
# lifetime (s): radiative only, then with black-body radiation
t0 = rb.getStateLifetime(n, 0, 0.5)
t300 = rb.getStateLifetime(n, 0, 0.5, temperature=300,
                           includeLevelsUpTo=n + 25)
# n, l, j, mj, n', l', j', q, E (V/m)  (rad/s)
rabi = rb.getRabiFrequency2(n, 0, 0.5, 0.5, n, 1, 1.5, 1, 0.1)

print(f"{abs(f)/1e9:.2f} GHz, {abs(d):.0f} e a0")
print(f"{t0*1e6:.0f} us at 0 K, {t300*1e6:.0f} us at 300 K")
print(f"Rabi at 1 mV/cm: {abs(rabi)/6.2832/1e6:.2f} MHz")
```

Output

```
30.80 GHz, 1450 e a0
129 us at 0 K, 62 us at 300 K

Rabi at 1 mV/cm:  1.85 MHz
```

- **ARC**: Alkali Rydberg Calculator, open-source Python
- The tables in this lecture came from calls like these

```
pip install

ARC-Alkali-Rydberg-Calculator
```

Šibalić *et al.*, Comput. Phys. Commun. 220, 319 (2017)

Hands-on: a Stark map in ten lines


```python
import numpy as np
from arc import Rubidium87, StarkMap

calc = StarkMap(Rubidium87())
# state: n, l, j, mj; then basis: nMin, nMax, maxL
calc.defineBasis(40, 2, 2.5, 2.5, 37, 43, 20,
                 progressOutput=False)
fields = np.linspace(0, 2000, 60)       # V/m
calc.diagonalise(fields, progressOutput=False)
calc.plotLevelDiagram(units="GHz", highlightState=True)

# energies are absolute: centre the view on the state
e0 = calc.y[0][calc.indexOfCoupledState]
calc.ax.set_ylim(e0 - 110, e0 + 110)
calc.fig.savefig("stark_map.png", dpi=200)
```

About a minute on a multi-core desktop, longer on a laptop. The basis size sets the cost: try `maxL` = 10.

- **Gotcha:** ARC returns absolute energies, so a plot centred on 0 is blank
- Inputs are in **V/m** even though labs quote V/cm

Exercise. Find the field at which $40D_{5/2}$ first meets the manifold. How does it scale with n? (Inglis–Teller predicts n^{-5}.)

The atom as a resonant dipole antenna

Compare a short dipole and a resonant two-level atom:

- Short dipole, matched load:
  $A_e = \dfrac{3\lambda^2}{8\pi} \quad (G = 3/2)$
- Lossless atom on resonance:
  $\sigma_0 = \dfrac{3\lambda^2}{2\pi} = 4A_e$

For the antenna engineer

The factor of 4 is familiar: a lossless resonant dipole with a shorted load scatters $4A_e$. An atom has no load. It only re-radiates.

Neither aperture depends on the physical size: the atom's cross-section is set by λ, even though the atom is $10^{-4}\lambda$ across.

An atom is the ultimate electrically small antenna: $ka \sim 10^{-4}$ and a matching network provided by nature. The catch is its extremely narrow radiative line, on the next slide.

Why $\sigma_0 = 3\lambda^2/2\pi$: the dipole cancels

1. Scattered power: photons per second $\times$ energy,

$$P = \hbar\omega\,\Gamma\,\rho_{ee}$$

2. Weak resonant drive: $\rho_{ee} = \Omega^2/\Gamma^2$, with $\Omega = dE/\hbar$
3. Incident intensity: $I = \frac{1}{2}\epsilon_0 c E^2$
4. Cross-section $\sigma = P/I$:

$$\sigma = \frac{2\omega d^2}{\epsilon_0 \hbar c\,\Gamma}$$

5. Insert $\Gamma = \omega^3 d^2/3\pi\epsilon_0\hbar c^3$:

$$\sigma_0 = \frac{6\pi c^2}{\omega^2} = \frac{3\lambda^2}{2\pi}$$

Both d and $\hbar$ have vanished. A bigger dipole absorbs more strongly **and** radiates faster, and the two exactly compensate.

For the antenna engineer

The same result as for any lossless resonant dipole: aperture is set by λ and the pattern, not by the structure.

A very high-Q antenna

Radiative decay on a single transition:

$$A = \frac{\omega^3 d^2}{3\pi\epsilon_0 \hbar c^3}, \qquad Q_{\mathrm{rad}} = \frac{\omega}{A}$$

Real linewidth is much larger than A, which reduces the cross-section:

$$\sigma = \sigma_0 \frac{A}{\Gamma_{\mathrm{tot}}}$$

Back of the envelope

Rb $50S \to 50P_{3/2}$, 30.8 GHz, $d = 1450\, ea_0$:
$A \approx 4.6\,\mathrm{s}^{-1}$, $Q_{\mathrm{rad}} \approx 4 \times 10^{10}$
$\sigma_0 \approx 45\,\mathrm{mm}^2$
with $\Gamma_{\mathrm{tot}}/2\pi = 1\,\mathrm{MHz}$: $\sigma \approx 30\,\mu\mathrm{m}^2$

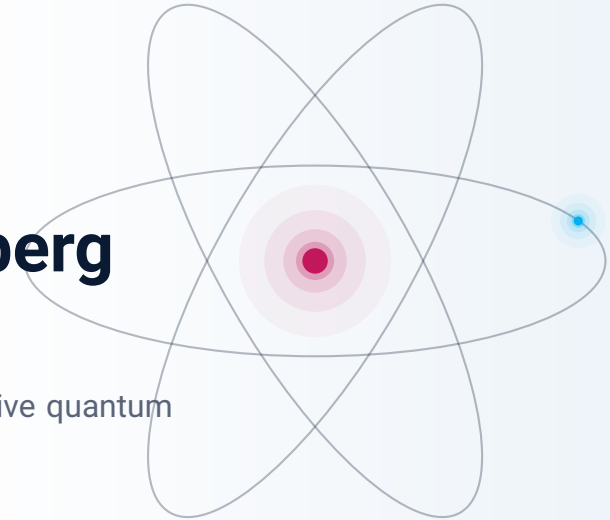
The atom barely absorbs a radio wave. We do not need it to: **a laser reads out the atomic state**, so the sensor is not a power-extracting antenna.

Chu's limit concerns matched power transfer: here bandwidth comes from the optical readout, not Q_{rad}.

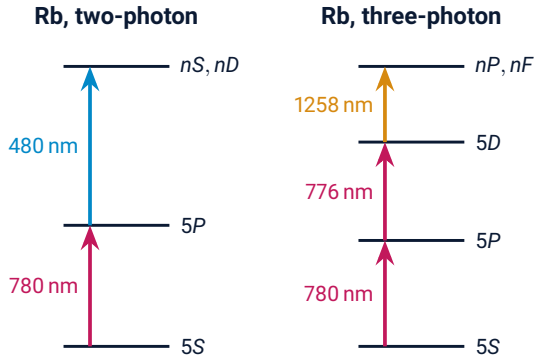
02

Controlling Rydberg atoms with light

From two-photon ladders to collective quantum states



Reaching Rydberg states with lasers



- Direct excitation needs UV (297 nm for Rb)
- Two photons: visible blue laser, high power
- Three photons: all near-infrared, diode lasers, partial **Doppler cancellation**
- Cs is similar: 852 + 509 nm

Coupling a Rydberg state is weak: optical dipoles scale as $n^{-3/2}$. This is why the coupling laser needs power.

From a dipole in a field to the Rabi frequency

Atom with levels $|g\rangle$, $|e\rangle$ in $E(t) = E_0 \cos\omega t$:

$$H = \hbar\omega_0 \, |e\rangle\langle e| - dE_0 \cos\omega t \, \big(\, |e\rangle\langle g| + |g\rangle\langle e| \, \big)$$

Move to the frame rotating at ω, $|e\rangle \to e^{-i\omega t} \, |e\rangle$. Then

$$\cos\omega t \, e^{i\omega t} = \tfrac{1}{2}\big(1 + e^{2i\omega t}\big)$$

The 2ω term averages out if $\Omega \ll \omega$ (**rotating-wave approximation**):

$$H = \hbar \begin{pmatrix} 0 & \Omega/2 \\ \Omega/2 & -\Delta \end{pmatrix}, \quad \Omega = \frac{dE_0}{\hbar}, \; \Delta = \omega - \omega_0$$

For the antenna engineer

The same step as going to the baseband: keep the slowly varying envelope and drop the term at twice the carrier.

Sign of d absorbed in the phase of Ω.

Back of the envelope

Optical: $\Omega/2\pi \sim$ MHz against $\omega/2\pi \sim$ 400 THz. Rydberg–Rydberg: MHz against 30 GHz. RWA is excellent in both.

Rabi oscillations, derived

1. Rewrite the Hamiltonian with Pauli matrices:

$$H = -\frac{\hbar\Delta}{2} + \frac{\hbar}{2}\left(\Omega\sigma_x + \Delta\sigma_z\right) = -\frac{\hbar\Delta}{2} + \frac{\hbar\tilde{\Omega}}{2}\,\mathbf{n}\cdot\boldsymbol{\sigma}$$

with $\tilde{\Omega} = \sqrt{\Omega^2 + \Delta^2}$, $\mathbf{n} = (\Omega, 0, \Delta)/\tilde{\Omega}$

2. Since $(\mathbf{n}\cdot\boldsymbol{\sigma})^2 = \mathbb{1}$, the exponential is simple:

$$e^{-i\theta\,\mathbf{n}\cdot\boldsymbol{\sigma}/2} = \cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\,\mathbf{n}\cdot\boldsymbol{\sigma}$$

3. Start in $|g\rangle$, take $\theta = \tilde{\Omega}t$:

$$P_e(t) = \frac{\Omega^2}{\Omega^2 + \Delta^2}\sin^2\frac{\tilde{\Omega}t}{2}$$

- On resonance: full transfer at $\Omega t = \pi$
- Off resonance: faster, weaker. This is the **Lorentzian** of a resonator
- A field can be measured by **counting oscillations**: no calibration needed

Geometrically, the state precesses on the Bloch sphere around the axis **n**.

Adding decay: where the Lorentzian comes from

1. Open systems obey the Lindblad equation:

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \tfrac{1}{2}\{L_k^\dagger L_k, \rho\} \right)$$

2. Spontaneous emission, $L = \sqrt{\Gamma}\,|g\rangle\langle e|$, and a weak drive, $\rho_{gg} \approx 1$:

$$\dot{\rho}_{eg} = -\left(\tfrac{\Gamma}{2} - i\Delta\right)\rho_{eg} + \tfrac{i\Omega}{2}$$

3. Steady state, $\dot{\rho}_{eg} = 0$:

$$\rho_{eg} = \frac{i\Omega/2}{\Gamma/2 - i\Delta}$$

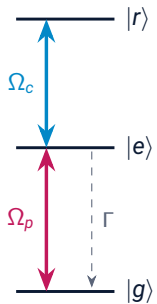
- Im part: absorption, a Lorentzian of FWHM Γ
- Re part: dispersion
- Coherence decays at $\Gamma/2$, population at Γ

For the antenna engineer

A driven, damped *LC* resonator in the time-harmonic limit: amplitude response $1/(\Gamma/2 - i\Delta)$.

Every spectrum in this lecture comes from solving such linear equations.

Three levels: the dark state



On two-photon resonance a superposition decouples from $|e\rangle$:

$$|D\rangle \propto \Omega_c |g\rangle - \Omega_p |r\rangle$$

- Excitation amplitudes via the two paths **cancel**: no population in $|e\rangle$, no scattering
- The probe passes through: **electromagnetically induced transparency**
- $|r\rangle$ is admixed, so the transparency is a direct **readout of the Rydberg state**

Harris, Physics Today 50(7), 36 (1997); Mohapatra *et al.*, PRL 98, 113003 (2007)

EIT, step 1: the Hamiltonian

Apply the rotating frame twice. Basis $\{|g\rangle, |e\rangle, |r\rangle\}$:

$$H = \hbar \begin{pmatrix} 0 & \Omega_p/2 & 0 \\ \Omega_p/2 & -\Delta & \Omega_c/2 \\ 0 & \Omega_c/2 & -\delta \end{pmatrix}$$

$\Delta = \Delta_p$: one-photon detuning
$\delta = \Delta_p + \Delta_c$: two-photon detuning

Decay: Γ from $|e\rangle$ (fast, 6 MHz), and γ of the $|g\rangle - |r\rangle$ coherence (slow, kHz to MHz).

Structure of the matrix: $|g\rangle$ and $|r\rangle$ never couple directly. Everything goes through $|e\rangle$, the only leaky state.

Idea: find a state that has no component on $|e\rangle$ and is not driven into it. Then nothing scatters.

EIT, step 2: finding the dark state

Look for $H|D\rangle = 0$ with $\delta = 0$ and $|D\rangle = a|g\rangle + b|e\rangle + c|r\rangle$:

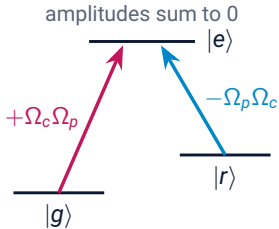
$$\frac{\Omega_p}{2} b = 0, \quad \frac{\Omega_p}{2} a - \Delta b + \frac{\Omega_c}{2} c = 0, \quad \frac{\Omega_c}{2} b = 0$$

So $b = 0$ and $\Omega_p a + \Omega_c c = 0$:

$$|D\rangle = \frac{\Omega_c |g\rangle - \Omega_p |r\rangle}{\sqrt{\Omega_p^2 + \Omega_c^2}}$$

Independent of Δ and Γ. Orthogonal **bright** state:

$$|B\rangle = \frac{\Omega_p |g\rangle + \Omega_c |r\rangle}{\sqrt{\Omega_p^2 + \Omega_c^2}}$$



Two excitation paths into $|e\rangle$ with **opposite amplitudes**: destructive interference. For a weak probe $|D\rangle \approx |g\rangle$: the ground state itself becomes transparent.

EIT, step 3: solving for the coherence

1. Weak probe, so $\rho_{gg} \approx 1$. Only two coherences matter:

$$\dot{\rho}_{eg} = -\left(\frac{\Gamma}{2} - i\Delta\right)\rho_{eg} + \frac{i\Omega_p}{2} + \frac{i\Omega_c}{2}\rho_{rg}$$

$$\dot{\rho}_{rg} = -(\gamma - i\delta)\rho_{rg} + \frac{i\Omega_c}{2}\rho_{eg}$$

2. Steady state of the second equation:

$$\rho_{rg} = \frac{i\Omega_c/2}{\gamma - i\delta}\,\rho_{eg}$$

3. Substitute into the first:

$$\rho_{eg} = \frac{i\Omega_p/2}{\dfrac{\Gamma}{2} - i\Delta + \dfrac{\Omega_c^2/4}{\gamma - i\delta}}$$

Compare with the two-level result: the only change is one added term in the denominator.

For the antenna engineer

A parallel *LC* trap in series with a lossy resonator: the $\frac{\Omega_c^2/4}{\gamma - i\delta}$ branch cuts a notch into the passband.

EIT, step 3 continued: the susceptibility

1. Polarisation of N atoms per unit volume, and definition of χ:

$$P = 2Nd\,\rho_{eg} = \epsilon_0 \chi E, \qquad \Omega_p = \frac{dE}{\hbar}$$

2. The probe field cancels:

$$\chi = \frac{Nd^2}{\epsilon_0 \hbar} \frac{i}{\frac{\Gamma}{2} - i\Delta + \frac{\Omega_c^2/4}{\gamma - i\delta}}$$

3. Propagation: $E(z) \propto e^{ik(1+\chi/2)z}$, so intensity decays at the rate $k\,\mathrm{Im}\,\chi$

Limits:

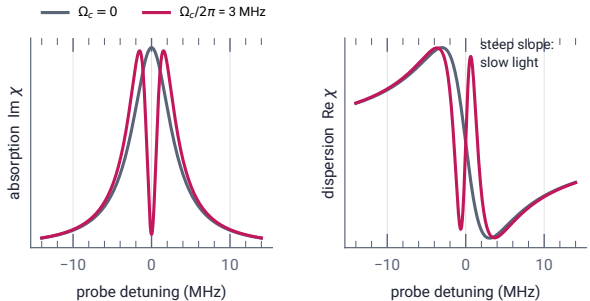
- $\Omega_c = 0$: a Lorentzian
- On resonance, $\delta = \Delta = 0$:

$$\chi \to \frac{Nd^2}{\epsilon_0 \hbar} \frac{i}{\Gamma/2 + \Omega_c^2/4\gamma}$$

- Ideal $\gamma \to 0$: $\chi \to 0$. **Perfect transparency**

Residual absorption measures γ: Rydberg decoherence is visible directly in EIT contrast.

EIT, step 4: the spectrum and slow light



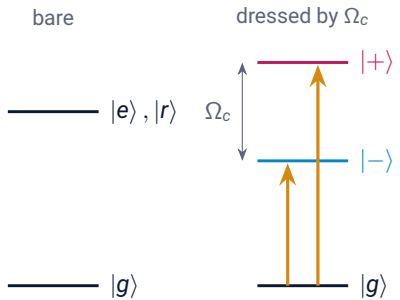
Near resonance ($\gamma \to 0$, $\delta = \Delta$), χ is linear:

$$\chi' \approx \frac{Nd^2}{\epsilon_0 \hbar} \frac{4\Delta}{\Omega_c^2}, \quad n_g \approx \frac{2\omega N d^2}{\epsilon_0 \hbar \, \Omega_c^2}$$

Back of the envelope

Cold Rb, $N = 10^{11}\,\mathrm{cm}^{-3}$, $\Omega_c/2\pi = 5\,\mathrm{MHz}$:
$n_g \approx 3 \times 10^5$, so $v_g \approx 1\,\mathrm{km/s}$.

EIT, step 5: the dressed-state view



Diagonalise the strong $|e\rangle - |r\rangle$ block ($\Delta = \delta = 0$):

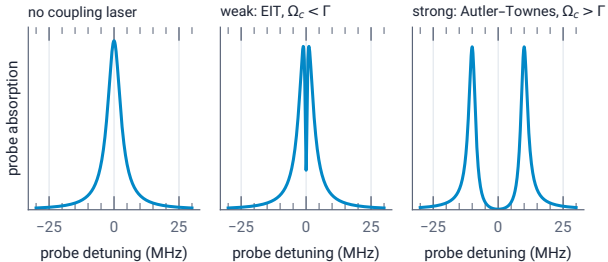
$$|\pm\rangle = \frac{|e\rangle \pm |r\rangle}{\sqrt{2}}, \quad E_\pm = \pm\frac{\hbar\Omega_c}{2}$$

Two lines, each carrying half the absorption:

- $\Omega_c \gg \Gamma$: well separated: **Autler–Townes**, no interference needed
- $\Omega_c \lesssim \Gamma$: lines overlap, and their amplitudes have **opposite sign** at $\Delta = 0$: a Fano-type dip, **EIT**

The same atom, the same formula: EIT is interference, Autler–Townes is level splitting. RF electrometry lives in the second regime.

EIT and Autler–Townes: two regimes



EIT, $\Omega_c < \Gamma$

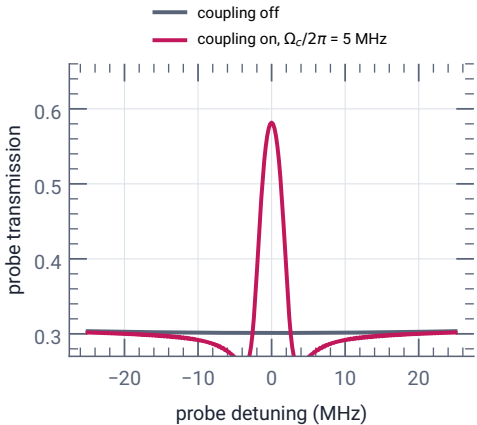
- Interference of two paths; width $\sim \Omega_c^2/\Gamma$

Autler–Townes, $\Omega_c > \Gamma$

- **Dressed** atom: two lines, split by Ω_c

Stationary atoms, $\Gamma/2\pi = 6.07$ MHz; Autler & Townes, Phys. Rev. 100, 703 (1955)

Hot atoms: Doppler and how to live with it



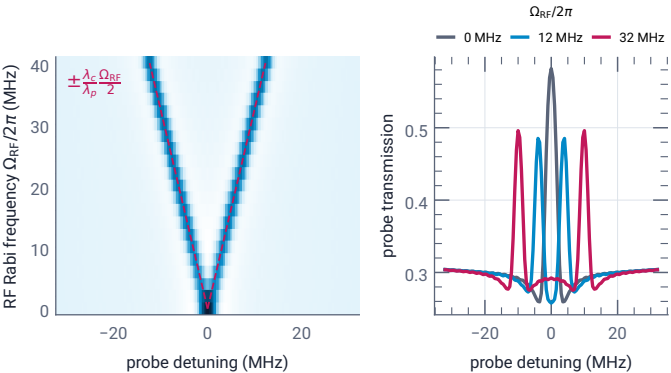
- Thermal velocity $\sim$ 240 m/s: Doppler width 500 MHz at 780 nm, about 100$\times$ natural width
- **Counter-propagating** beams: an atom sees the Doppler shifts partly cancel,

$$\delta_2 = (k_c - k_p)\, v$$

- Only one velocity class gives the narrow feature

Three-photon schemes can make $\sum_i \mathbf{k}_i \approx 0$, which removes residual Doppler.

Adding a microwave field



four-level ladder, Doppler-averaged over 300 K atoms, $\Omega_c/2\pi = 5$ MHz

- An RF field couples $|r\rangle \leftrightarrow |r'\rangle$
- Dressing splits the EIT line
- The splitting is linear in E

Probe scan in vapour:
splitting $= \dfrac{\lambda_c}{\lambda_p}\dfrac{\Omega_{\mathrm{RF}}}{2\pi}$

Simulation slope: 0.615. Theory λ_c/λ_p: 0.615

Why that factor? Velocity selection

Counter-propagating beams, atom velocity v:

- Intermediate state resonance: $\Delta_p - k_p v = 0$
- Dressed Rydberg resonance: $(\Delta_p - k_p v) + (\Delta_c + k_c v) = \pm\Omega/2$

With $\Delta_c = 0$ the two conditions give $v = \pm\Omega/2k_c$, so

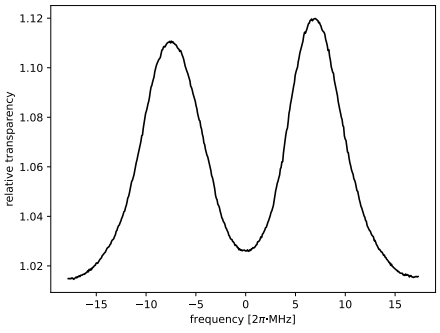
$$\Delta_p = \pm\frac{k_p}{k_c}\frac{\Omega}{2}$$

A probe scan selects a **different velocity class** at each detuning, which rescales the frequency axis.

Scanning the **coupling** laser instead selects $v = 0$, and the splitting is just Ω.

A good exam question: in electrometry papers, always ask which laser was scanned.

This is already an absolute field meter



measured Autler–Townes doublet, our lab

$$E = \frac{h}{d}\frac{\Omega}{2\pi}$$

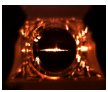
- A frequency measurement gives the field
- h is exact in the SI, d is computed
- **No calibration antenna**: traceable to the SI by construction

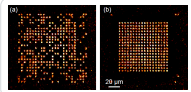
Bridge to Lecture 2: first demonstrated in a vapour cell by Sedlacek *et al.*, Nat. Phys. 8, 819 (2012). It limits at ∼mV/cm, where the doublet stops being resolved. Getting below that is where the receiver story begins.

Three experimental platforms

	Hot vapour	Cold cloud (MOT)	Tweezer array
Temperature	300–360 K	∼100 µK	∼10 µK
Atoms	10^9–10^{12} cm^{-3}	10^8 atoms in a cloud	1 per trap, spacing µm
Doppler	500 MHz	negligible	negligible
Operation	continuous	pulsed, trap cycle	pulsed, single-atom control
Hardware	glass cell, heater	vacuum, cooling lasers, coils	+ optical tweezers, SLM/AOD
Strength	simple, robust, portable	coherence, clean spectra	individual addressing
Use	**sensing, receivers**	memories, conversion, metrology	**quantum computing**



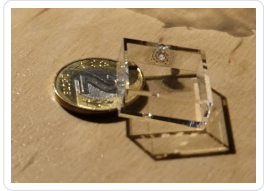




Vapour cells in numbers

Cell temperature	Rb density (cm^{-3})
20 °C	7×10^{9}
40 °C	6×10^{10}
58 °C	3×10^{11}
85 °C	2×10^{12}

- Density doubles every 7–9 K
- Rb vapour pressure set by the **coldest spot**
- Optimum is a trade-off: signal against absorption and interactions



Back of the envelope

Beam 200 µm, $v \approx 240$ m/s: atoms cross in ~ 1 µs.

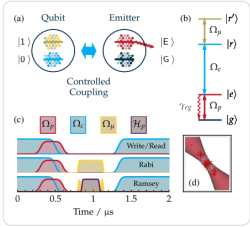
Transit broadening of a few hundred kHz, but it also **replenishes** atoms: no trapping for continuous operation.

One photon, many atoms: collective excitations

A single excitation shared by N atoms:

$$|W_{\mathbf{k}}\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{i\mathbf{k}\cdot\mathbf{r}_j} |g \ldots r_j \ldots g\rangle$$

- Coupling enhanced: $\Omega \to \sqrt{N}\,\Omega$
- The phase pattern $e^{i\mathbf{k}\cdot\mathbf{r}}$ is a **spin wave**
- On readout atoms emit in phase, in one direction, set by **phase matching**



N. L. R. Spong *et al.*, PRL 127, 063604 (2021)

For the antenna engineer

A **phased array**: the spin-wave phase is the excitation, and the array factor gives the emission pattern.

Why collective emission is directional

1. Atom j emits with the phase written into it. Field in direction $\mathbf{k}'$:

$$E \propto \sum_{j=1}^{N} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_j}$$

2. Square it. Diagonal and off-diagonal terms:

$$I \propto N + \sum_{j\neq l} e^{i\Delta\mathbf{k}\cdot(\mathbf{r}_j-\mathbf{r}_l)}$$

3. Average over random atom positions, with density $\rho(\mathbf{r})$:

$$I \propto N + N(N-1)\,|\tilde{\rho}(\Delta\mathbf{k})|^2$$

- First term: incoherent, in all directions
- Second term: peak N^2 at $\Delta\mathbf{k} = 0$, with angular width $\sim \lambda/L$

For the antenna engineer

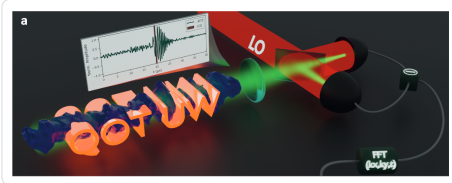
$\tilde{\rho}$ is exactly an **array factor**: the Fourier transform of the aperture. Large N gives a beam.

This is why memories read out in a defined mode.

Quantum memories for light

- Slow the light with EIT, switch off the coupling: the pulse becomes a spin wave
- Switch on again, and the photon comes back
- Key tool for **quantum repeaters**
- Our lab: many spatial modes at once, using the wavevector of the spin wave

Why this matters for sensing: a stored spin wave is a coherent **probe** that can be exposed to a microwave, then read out with light.



3D tomography of a stored excitation

Mazelanik *et al.*, Commun. Phys. 6, 165 (2023)

Parniak *et al.*, Nat. Commun. 8, 2140 (2017)

Stopping light: the dark-state polariton

1. Group index from the slope of χ' (EIT step 4):

$$\frac{c}{v_g} \approx n_g = \frac{2\omega N d^2}{\epsilon_0 \hbar \Omega_c^2} \equiv \tan^2\theta$$

2. A combined light–atom excitation propagates without loss:

$$\hat{\Psi} = \cos\theta\,\hat{E} - \sin\theta\,\sqrt{N_{\mathrm{tot}}}\,\hat{\sigma}_{gr}$$

3. Its velocity follows the control field:

$$v_g = c\cos^2\theta = \frac{c}{1+\tan^2\theta}$$

4. Ramp $\Omega_c \to 0$: $\theta \to \pi/2$, and $v_g \to 0$

The pulse does not stop by being absorbed. It is **rotated** from photons into a spin wave, and Ω_c turns the knob.

Reverse the ramp and the photon returns in the same mode.

Fleischhauer & Lukin, PRL 84, 5094 (2000)

Motional dephasing, derived

1. A moving atom shifts the spin-wave phase:

$$e^{i\mathbf{k}\cdot\mathbf{r}_j} \to e^{i\mathbf{k}\cdot\mathbf{r}_j}\, e^{ikv_j t}$$

2. Readout sums over the Maxwell distribution, width $\sigma_v = \sqrt{k_B T/m}$:

$$\left\langle e^{ikvt} \right\rangle = \int \frac{dv}{\sqrt{2\pi}\sigma_v} e^{-v^2/2\sigma_v^2} e^{ikvt}$$

3. The Fourier transform of a Gaussian is a Gaussian:

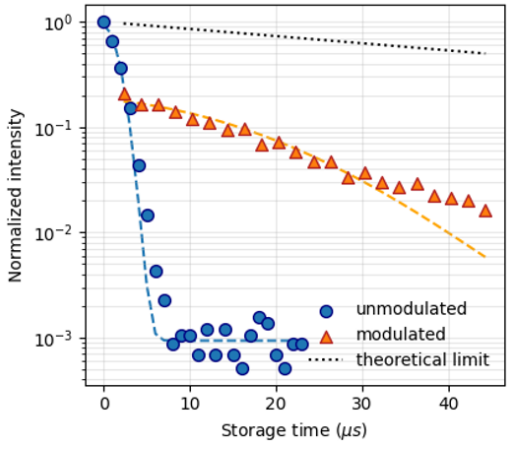
$$\left\langle e^{ikvt} \right\rangle = e^{-t^2/2\tau^2}, \qquad \tau = \frac{1}{k\sigma_v}$$

Back of the envelope

Rb, 100 µK: $\sigma_v \approx 0.1\,\mathrm{m/s}$.
Rydberg spin wave, beams opposed: $k = k_c - k_p \approx 5 \times 10^6\,\mathrm{m}^{-1}$.
$\tau \approx 2\,\mu\mathrm{s}$, far below the 100 µs Rydberg lifetime.

k is the problem, not the Rydberg state. So reduce k: that is the next slide.

Fighting motional dephasing



- Moving atoms scramble the phase $\mathbf{k} \cdot \mathbf{r}_j$: Rydberg spin waves live for only a few μs, since large k arises from the two-photon mismatch
- Remedy: an **ac-Stark lattice** moves the spin wave into a low-k "frozen" state, and back
- Lifetime extended by an order of magnitude

Kurzyna *et al.*, Quantum 8, 1431 (2024)

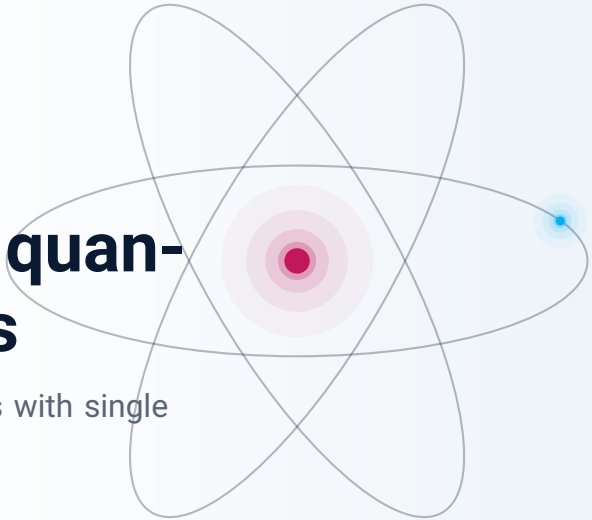
Break

Part 3 next: what happens when Rydberg atoms talk to each other

03

Interactions and quantum technologies

Blockade, gates and nonlinear optics with single photons

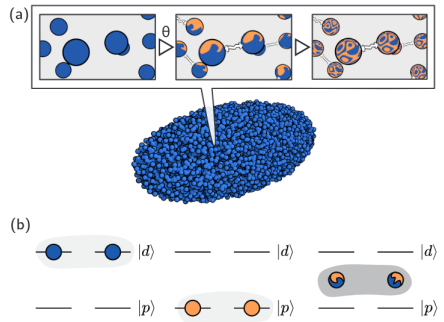


Two dipoles, a few micrometres apart

Dipole–dipole interaction:

$$V_{dd} = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{d}_1 \cdot \mathbf{d}_2 - 3(\mathbf{d}_1 \cdot \hat{\mathbf{R}})(\mathbf{d}_2 \cdot \hat{\mathbf{R}})}{R^3}$$

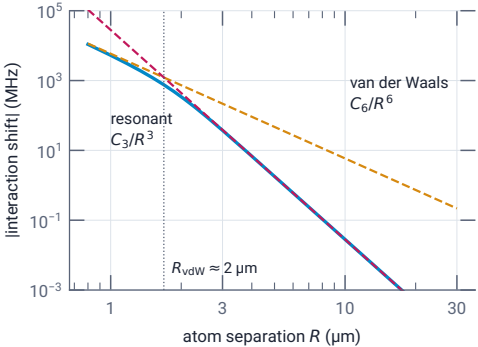
- Same physics as near-field coupling of two antennas
- Rydberg d is 10^3 times larger than a ground-state optical dipole, so V_{dd} is 10^6 times larger



Back of the envelope

$d \approx 1000\, ea_0$ at $R = 5\,\mu\text{m}$: $V_{dd}/h \approx 8\,\text{MHz}$. Larger than the MHz linewidths and Rabi frequencies we use.

Resonant or van der Waals?



two-channel model: $60S\,60S \leftrightarrow 59P\,60P$

Pair state coupled to a neighbour with energy defect δ:

$$\begin{pmatrix} 0 & V \\ V & \delta \end{pmatrix}, \quad V = \frac{C_3}{R^3}$$

- $V \gg \delta$: shift $\pm C_3/R^3$. Resonant, long range
- $V \ll \delta$: shift $V^2/\delta = C_6/R^6$, with $C_6 \propto n^{11}$

A **Förster resonance** ($\delta \to 0$) keeps $1/R^3$ at all distances.

Full sum: $|C_6| = 139$ GHz µm^6 for $60S$.

Where C_6 and n^{11} come from

Second-order perturbation theory for $|rr\rangle$:

$$\Delta E = \sum_{p,p'} \frac{|\langle pp'| V_{dd} |rr\rangle|^2}{E_{rr} - E_{pp'}} = \frac{C_6}{R^6} \quad \text{(up to convention)}$$

Each matrix element $\sim d^2/4\pi\epsilon_0 R^3$, so

$$C_6 \sim \frac{d^4}{(4\pi\epsilon_0)^2\,\delta}$$

Scaling: $d \propto n^2$, defect $\delta \propto n^{-3}$:

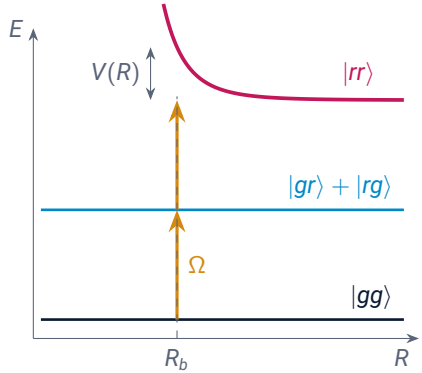
$$C_6 \propto \frac{(n^2)^4}{n^{-3}} = n^{11}$$

Compare $C_3 \propto d^2 \propto n^4$ on a Förster resonance.

For the antenna engineer

Two near-field-coupled resonators, detuned by δ: the coupling only shifts them at second order, V^2/δ. Tune them together and the coupling becomes first order.

The Rydberg blockade



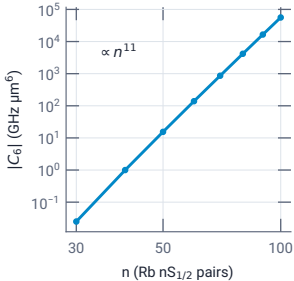
- One atom in $|r\rangle$ shifts its neighbours out of resonance
- Within R_b, **only one excitation is allowed**

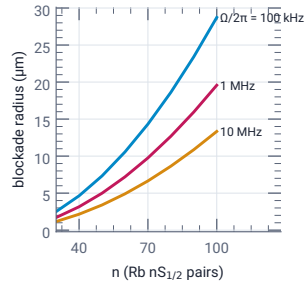
$$R_b = \left(\frac{|C_6|}{\hbar\Omega}\right)^{1/6}$$

- Sixth root: R_b is very forgiving, about 5 µm to 20 µm
- One atom switches another

Jaksch *et al.*, PRL 85, 2208 (2000); Lukin *et al.*, PRL 87, 037901 (2001)

Blockade radius in numbers





Back of the envelope

70S: $|C_6| = 860\,\text{GHz}\,\mu\text{m}^6$, $\Omega/2\pi = 1\,\text{MHz}$:

$R_b = (860\,000)^{1/6}\,\mu\text{m} \approx 9.8\,\mu\text{m}$

- Typical tweezer spacing is 3–10 µm
- In a cloud, a blockade sphere contains many atoms

Blockade, derived: two atoms

Both driven by Ω, interaction V only in $|rr\rangle$:

$$H/\hbar = \frac{\Omega}{2} \sum_{i=1,2} \left(|r\rangle\langle g|_i + \text{h.c.} \right) + V\,|rr\rangle\langle rr|$$

Symmetric and antisymmetric states:

$$|W_\pm\rangle = \frac{|gr\rangle \pm |rg\rangle}{\sqrt{2}}$$

$|W_-\rangle$ decouples. In $\{|gg\rangle, |W_+\rangle, |rr\rangle\}$:

$$H/\hbar = \begin{pmatrix} 0 & \Omega/\sqrt{2} & 0 \\ \Omega/\sqrt{2} & 0 & \Omega/\sqrt{2} \\ 0 & \Omega/\sqrt{2} & V \end{pmatrix}$$

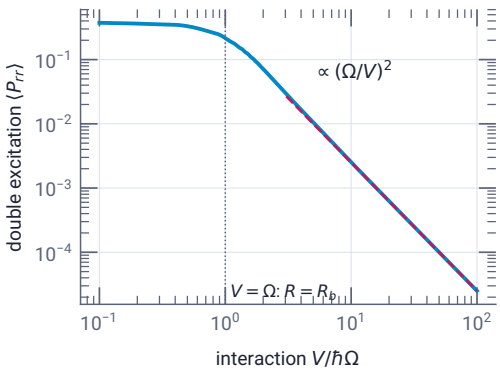
The $\sqrt{2}$ is simple counting: $\langle W_+|\, H\, |gg\rangle = 2 \times \frac{1}{\sqrt{2}} \times \frac{\Omega}{2}$.

If $V \gg \Omega$, $|rr\rangle$ is out of reach. What is left is a two-level system

$$|gg\rangle \leftrightarrow |W_+\rangle\,, \quad \Omega_{\text{eff}} = \sqrt{2}\,\Omega$$

For N atoms the same counting gives $\sqrt{N}\,\Omega$.

Blockade, derived: how perfect is it?



solid: exact time average; dashed: formula

Off-resonant admixture of $|rr\rangle$ from $|W_+\rangle$ has amplitude $\approx (\Omega/\sqrt{2})/V$. $|W_+\rangle$ is occupied half of the time on average, so

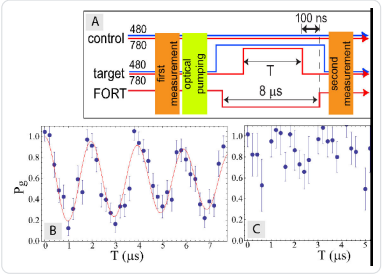
$$\langle P_{rr} \rangle \approx \frac{\Omega^2}{4V^2}$$

- Falls quadratically with V/Ω
- In distance: $\propto (R/R_b)^{12}$, a very sharp edge

Back of the envelope

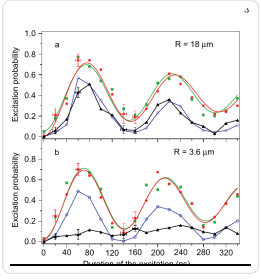
Gate-quality blockade, $P_{rr} < 10^{-3}$: $V/\Omega \gtrsim 16$, so $R \lesssim R_b/1.6$.

2009: blockade seen between two atoms



Wisconsin: atoms 10 µm apart. The control atom in $|r\rangle$ stops the target's Rabi flopping (C).

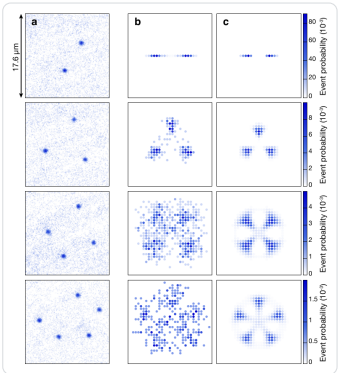
Urban *et al.*, Nat. Phys. 5, 110 (2009)



Palaiseau: two tweezers 4 µm apart. Double excitation suppressed; collective enhancement of single excitation.

Gaëtan *et al.*, Nat. Phys. 5, 115 (2009)

Blockade made visible: Rydberg crystals

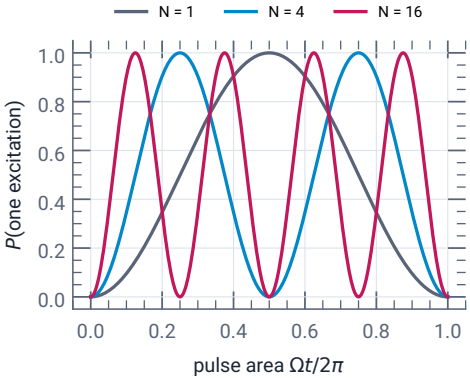


- **Munich:** a 2D Mott insulator in an optical lattice, excited to Rydberg states
- Quantum gas microscope images each excitation
- Excitations avoid each other within $R_b = 4.9$ μm
- Ordered ring-like patterns appear for 2 to 5 excitations

The blockade radius, measured with a **ruler**: a single image of a many-body quantum state.

Schauß *et al.*, Nature 491, 87 (2012)

Collective enhancement inside a blockade sphere



$$P(t) = \sin^2(\sqrt{N}\,\Omega t/2)$$

- N atoms, one excitation allowed: an effective two-level **superatom**
- Rabi frequency $\sqrt{N}\,\Omega$, the signature used to observe blockade
- A cloud smaller than R_b becomes a deterministic **single-photon source**

Dudin & Kuzmich, Science 336, 887 (2012)

For the antenna engineer

The ensemble behaves as a strongly non-linear element: a saturable absorber at the level of **one quantum**.

Hands-on: C_6 with pairinteraction


```python
import pairinteraction as pi

pi.Database.initialize_global_database(download_missing=True)

unit = "planck_constant * GHz * micrometer^6"
rabi_ghz = 1e-3                          # Omega / 2 pi = 1 MHz
for n in (50, 70, 100):
    ket = pi.KetAtom("Rb", n=n, l=0, j=0.5, m=0.5)
    c6 = pi.C6(ket, ket).get(unit=unit)
    r_b = (abs(c6) / rabi_ghz) ** (1 / 6)
    print(f"n = {n}: C6 = {c6:.1f}, R_b = {r_b:.1f} um")
```

Output

```
n = 50:  C6 = -15.4, R_b = 5.0 um
n = 70:  C6 = -868.6, R_b = 9.8 um

n = 100:  C6 = -56475.7, R_b = 19.6 um
```

`pip install pairinteraction` Weber *et al.*, J. Phys. B 50, 133001

Sanity check. For 70*S*, ARC gives -862.7 and pairinteraction -868.6. They use different quantum defects and bases, so a sub-1 % spread is a realistic error bar.

Sign. Here $V = -C_6/R^6$: *nS* pairs in Rb **repel**.

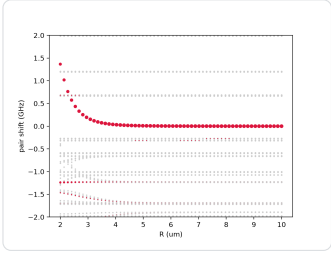
Hands-on: full pair potentials


```
pi.Database.initialize_global_database(download_missing=True)
ket = pi.KetAtom("Rb", n=60, l=0, j=0.5, m=0.5)

# single-atom basis, then pair basis within 3 GHz of |60S, 60S>
basis = pi.BasisAtom("Rb", n=(55, 65), l=(0, 3))
atom = pi.SystemAtom(basis)
atom.set_magnetic_field([0, 0, 1], unit="G").diagonalize()
e0 = 2 * atom.get_corresponding_energy(ket, unit="GHz")
pair = pi.BasisPair([atom, atom], energy=(e0 - 3, e0 + 3),
                    energy_unit="GHz")

# one pair system per distance; diagonalise all in parallel
distances = np.linspace(2, 10, 60)      # micrometres
systems = [pi.SystemPair(pair)
           .set_distance(r, unit="micrometer")
           .set_interaction_order(3) for r in distances]
pi.diagonalize(systems)
```

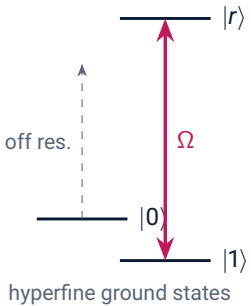
Plotting code is in `examples/`. Under a minute to run.



red: $|60S, 60S\rangle$ character

Perturbation theory fails at small R, where other pair states mix in. Diagonalising the pair Hamiltonian handles that.

A qubit in an atom, and the 2π trick



Qubit stored in long-lived ground states. The laser addresses only $|1\rangle \leftrightarrow |r\rangle$.

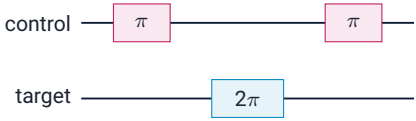
Resonant evolution, from $\sigma_x^2 = \mathbb{1}$:

$$U = e^{-i\Omega t\sigma_x/2} = \cos\frac{\Omega t}{2} - i\sin\frac{\Omega t}{2}\,\sigma_x$$

- π pulse: $|1\rangle \to -i\,|r\rangle$, $|r\rangle \to -i\,|1\rangle$
- 2π pulse: $U = -\mathbb{1}$, so $|1\rangle \to$ - $|1\rangle$

A full Rabi cycle returns the atom but flips the sign: the spin-$\frac{1}{2}$ property. This π phase is the gate's raw material.

The CZ gate, state by state



Input	What happens	Phase
$\|00\rangle$	nothing resonant	$+1$
$\|01\rangle$	target does 2π	-1
$\|10\rangle$	control $\pi + \pi$	-1
$\|11\rangle$	control $\pi + \pi$; target **blockaded**	-1

Without blockade, $|11\rangle$ would pick up $(-1)(-1) = +1$. Blockade removes the target's sign:

$$U = \mathrm{diag}(1, -1, -1, -1)$$

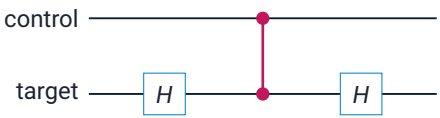
Up to single-qubit Z rotations, this is the **controlled-Z**:

$$U_{\mathrm{CZ}} = \mathrm{diag}(1, 1, 1, -1)$$

Interaction strength does not set the gate phase. As long as $V \gg \Omega$, the phase is **exactly** π: forgiving of atom distance.

Jaksch *et al.*, PRL 85, 2208 (2000)

From CZ to CNOT



Since $HZH = X$:

$$(\mathbb{1} \otimes H)\, U_{\mathrm{CZ}}\, (\mathbb{1} \otimes H) = U_{\mathrm{CNOT}}$$

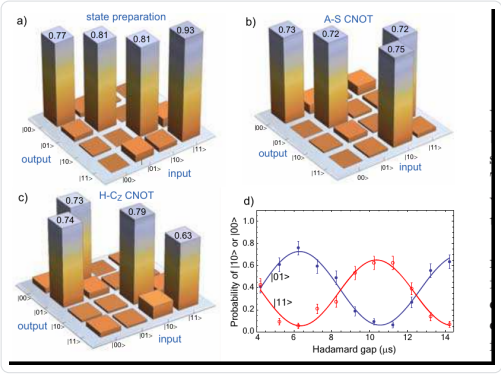
$$U_{\mathrm{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

In	Out
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$

Control in $|1\rangle$ applies Z to the target.
Hadamards turn Z into a bit flip.

Entanglement: $\frac{|0\rangle+|1\rangle}{\sqrt{2}}\,|0\rangle \xrightarrow{\text{CNOT}} \frac{|00\rangle+|11\rangle}{\sqrt{2}}$, a Bell state.

The first neutral-atom CNOT

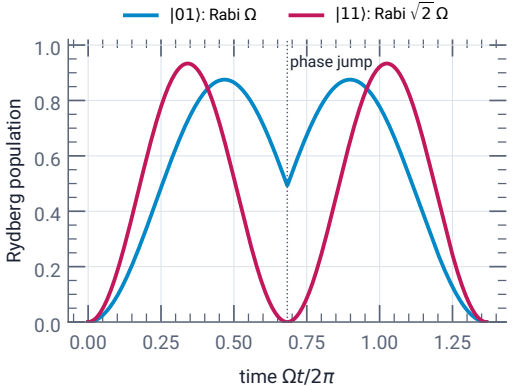


- **Wisconsin, 2010:** two Rb atoms in traps more than 8 μm apart
- Bars: measured truth tables
- CNOT fidelity 0.73 and 0.72 for two protocols
- Bell state with $F = 0.48 \pm 0.06$, higher after correcting for atom loss

From 73 % to above 99.5 % in 13 years: lasers, cooling and a better pulse design.

Isenhower *et al.*, PRL 104, 010503 (2010)

The modern gate: one global pulse pair



$\Delta/\Omega = 0.377, \Omega\tau = 4.293, \xi = 3.902$

Drive **both** atoms. No individual addressing:

- $|01\rangle$: Rabi frequency Ω
- $|11\rangle$: blockaded, so $\sqrt{2}\,\Omega$

Two detuned pulses, with a laser phase jump ξ between them, return both states to the ground manifold with phases ϕ_{01}, ϕ_{11}. A CZ needs

$$\phi_{11} = 2\phi_{01} - \pi$$

Check of the published parameters in our script: leakage $\sim 10^{-12}$ and $2\phi_{01} - \phi_{11} = 3.14159$.

Levine *et al.*, PRL 123, 170503 (2019)

How good are gates, and what limits them?

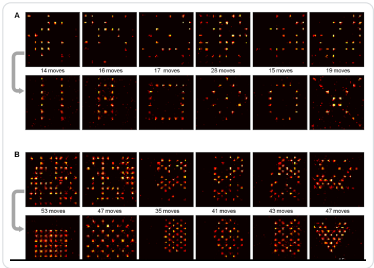
Year	CZ fidelity	Reference
2010	73 % (CNOT)	Isenhower, PRL 104, 010503
2019	97.4 %	Levine, PRL 123, 170503
2023	99.5 %	Evered, Nature 622, 268
2025	99.71 %	Tsai, PRX Quantum 6, 010331
2026	99.85 %	arXiv:2604.25987 (preprint)

- Rydberg decay during the gate
- Scattering from the intermediate state
- Laser phase and intensity noise
- Atomic motion, Doppler shifts

Back of the envelope

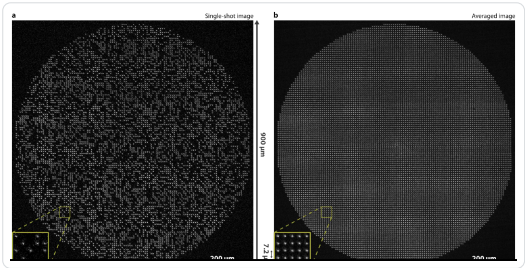
$\Omega/2\pi = 5\,\mathrm{MHz}$: gate time $2\tau \approx 270\,\mathrm{ns}$. Rydberg state occupied for about half of that, with $\tau_r \sim 150\,\mu\mathrm{s}$: decay error of order 10^{-3}. Fundamental fidelity limit now sits near 99.9 %.

Building arrays atom by atom



Defect-free arrays of up to ∼50 atoms, sorted by a moving tweezer

Barredo *et al.*, Science 354, 1021 (2016)

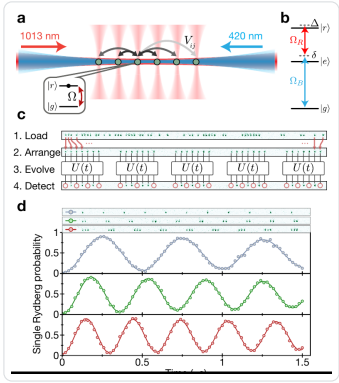


Over 6100 atoms in ∼12 000 sites; 12.6 s coherence

Manetsch *et al.*, Nature 647, 60 (2025)

Atoms are identical: no fabrication spread. Nine years, a factor of 100 in size.

A programmable quantum simulator



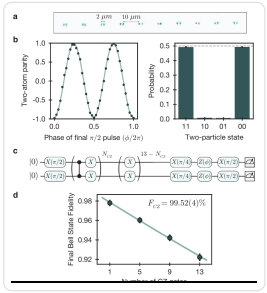
Blockade gives an Ising-type model:

$$\frac{H}{\hbar} = \sum_i \frac{\Omega}{2}\sigma_i^x - \sum_i \Delta n_i + \sum_{i<j} V_{ij} n_i n_j$$

- **Harvard/MIT:** up to 51 atoms in a chain
- Sweeping Δ prepares crystalline order
- Surprisingly persistent oscillations after a quench, later linked to quantum many-body scars

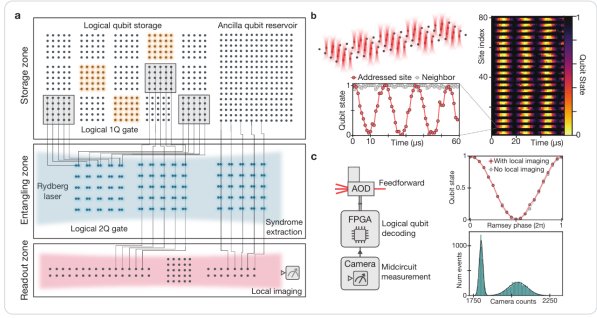
Bernien *et al.*, Nature 551, 579 (2017)

From gates to logical qubits



CZ fidelity 99.5 % on up to 60 atoms in parallel

Evered *et al.*, Nature 622, 268 (2023)



Zoned processor: storage, entangling and readout. Up to 48 logical qubits and 280 physical qubits

Bluvstein *et al.*, Nature 626, 58 (2024)

Single-photon nonlinear optics

Rydberg EIT maps atomic interaction onto light:

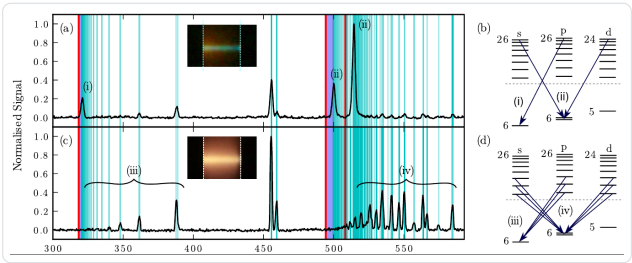
- A slow photon is a **Rydberg polariton**, part light, part atom
- Two polaritons within R_b blockade each other
- The medium turns absorbing or dispersive for the second photon
- Nonlinearity at the **level of one photon**

In ordinary materials photons barely notice each other without intense beams. In a Rydberg medium two photons are enough.

Milestones

- Photon antibunching in transmission: Peyronel *et al.*, Nature 488, 57 (2012)
- Attractive photons: Firstenberg *et al.*, Nature 502, 71 (2013)
- Photon–photon gate: Tiarks *et al.*, Nat. Phys. 15, 124 (2019)

Interactions in hot vapour: a phase transition in a cell

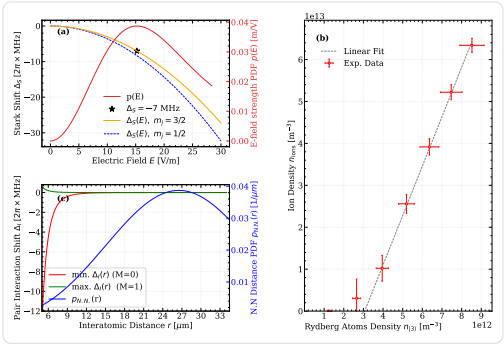


Fluorescence from low (top) and high (bottom) Rydberg occupation

Carr *et al.*, PRL 111, 113901 (2013)

- **Durham:** above a critical Rydberg density, **optical bistability** and critical slowing down
- A dilute thermal gas behaves as a driven many-body system
- Sharp response near the transition is useful for sensing: see Lecture 2

What causes the shift? Measuring it in vapour



Prokop, Kasza, Wasilewski, Parniak, arXiv:2607.15221 (2026)

- Interaction shift measured with a split-EIA balance method
- Result agrees with **Stark shifts from ions**, not van der Waals interactions
- Consistent with Weller *et al.*, PRA 94, 063820 (2016)

In a hot cell, ionisation matters: it sets the working point of any Rydberg receiver.

Cavity QED: where microwaves meet single atoms

Vacuum field of one mode, volume V:

$$E_{\mathrm{vac}} = \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}, \qquad g = \frac{dE_{\mathrm{vac}}}{\hbar}$$

Strong coupling: $g > \kappa, \gamma$.

- Haroche: Rydberg atoms cross a superconducting cavity, and **count single microwave photons** without destroying them
- One-atom maser (Walther): < 1 atom in the cavity at a time

Meschede *et al.*, PRL 54, 551 (1985); Raimond *et al.*, RMP 73, 565 (2001); Gleyzes *et al.*, Nature 446, 297 (2007)

Back of the envelope

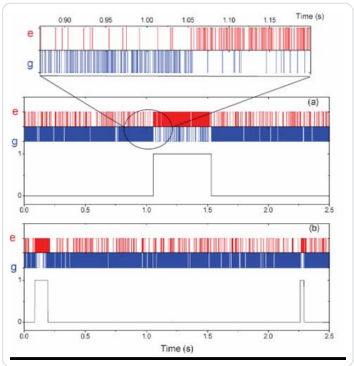
$f = 50\,\mathrm{GHz}$, $V = \lambda^3$ ($216\,\mathrm{mm}^3$), $d = 1000\,ea_0$:

$E_{\mathrm{vac}} \approx 3\,\mathrm{mV/m}$

$g/2\pi \approx 40\,\mathrm{kHz}$: one atom, one photon, with a superconducting cavity of $\kappa/2\pi \sim$ Hz–kHz.

Rydberg atoms are the **original** circuit-QED platform: microwave photons and an "artificial atom" with a macroscopic dipole. Today's superconducting qubits inherited the language.

Watching one photon live and die



- **ENS Paris:** circular Rydberg atoms cross a superconducting cavity one by one
- Each atom picks up a light shift if a photon is there, and is read out by a Ramsey sequence
- The photon is **not absorbed**: a quantum non-demolition measurement
- Quantum jumps of the field appear in real time. Cavity damping time 0.129 s

Lecture 2 asks how much of this survives at room temperature.

Gleyzes *et al.*, Nature 446, 297 (2007)

Cooling a field mode with atoms, derived

1. Cavity mode, loss κ to a thermal bath: $\dot{\bar{n}} = -\kappa(\bar{n} - \bar{n}_{\mathrm{th}})$
2. Atoms open a second exit, of width γ. Eliminating them (as for Purcell decay) gives the rate

$$\Gamma_{\mathrm{at}} = 4g^2 N/\gamma$$

3. This exit has an empty bath:

$$\dot{\bar{n}} = -\kappa(\bar{n} - \bar{n}_{\mathrm{th}}) - \Gamma_{\mathrm{at}}\bar{n}$$

4. Steady state, with $C = \Gamma_{\mathrm{at}}/\kappa = 4g^2 N/\kappa\gamma$:

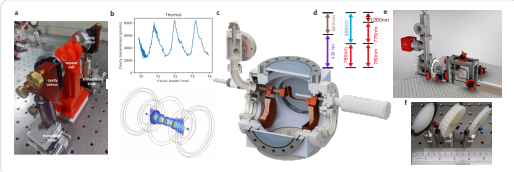
$$\bar{n} = \frac{\bar{n}_{\mathrm{th}}}{1 + C}$$

Light is at effectively zero temperature: at 384 THz, $\bar{n} \approx 0$ at 300 K. Extracting microwave photons as light is **refrigeration**.

For the antenna engineer

The same logic as "cold loads" in radiometry, and as active cooling of an amplifier input. Here the resistor is an atomic transition.

Cavity QED with ensembles: a quantum refrigerator?



QURA concept: Rydberg ensemble in a microwave cavity

- N atoms: $g \to \sqrt{N}g$
- Cooperativity $C = 4g^2/\kappa\gamma$
- Upconversion removes photons from the cavity mode, which can cool it by a factor $1 + C$

Goal of ERC QURA: a mode cooled by converting its photons to light, a room-temperature analogue of sideband cooling, with $C \sim 0.1$ to 100 estimated from measured single-pass conversion.

Coming in Lecture 2.

Summary

01 Giant atoms

n tunes everything: dipoles n^2, frequencies n^{-3}, lifetimes n^3. A ladder of narrow, polarisation-selective transitions from MHz to THz, with cross-section set by λ rather than size.

02 Control by light

EIT reads the Rydberg state optically. An RF field splits the line by an amount set by h and d. Ensembles behave as phased arrays, and spin waves store quantum states.

03 Interactions

$C_6 \propto n^{11}$ gives blockade over µm: quantum gates, single-photon nonlinearity, and in cavities, control of microwave photons.

Next lecture: how these atoms become receivers, transducers and photon counters. Try the ARC and pairinteraction scripts in `examples/`.

Further reading

Books and reviews

- T. F. Gallagher, *Rydberg Atoms*, CUP (1994)
- M. Saffman, T. G. Walker, K. Mølmer, RMP 82, 2313 (2010)
- C. S. Adams, J. D. Pritchard, J. P. Shaffer, J. Phys. B 53, 012002 (2020)
- A. Browaeys, T. Lahaye, Nat. Phys. 16, 132 (2020)
- G. Allinson *et al.*, *Rydberg receivers for space applications*, arXiv:2601.20631 (2026)

Software

- ARC: arc-alkali-rydberg-calculator.readthedocs.io
- pairinteraction: www.pairinteraction.org

Our work

- Publications: www.qodl.eu/publications

Thank you!

Quantum Optical Devices Lab, University of Warsaw

www.qodl.eu **qot.uw.edu.pl** **qura.space**

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